Chapter 12
On Linearly Constrained QRD-Based Algorithms

Shiunn-Jang Chern

Abstract The linearly constrained adaptive filtering (LCAF) technique has been extensively used in many engineering applications. In this chapter, we introduce the linearly constrained minimum variance (LCMV) filter, implemented using the linearly constrained recursive least squares (RLS) criterion, with the inverse QR decomposition (IQRD) approach. First, the direct form of recursively updating the constrained weight vector of LS solution based on the IQRD is developed, which is named as the LC-IQRD-RLS algorithm. With the IQRD approach, the parameters related to the Kalman gain are evaluated via Givens rotations and the LS weight vector can be computed without back-substitution. This algorithm is suitable to be implemented using systolic arrays with very large scale integration technology and DSP devices. For the sake of simplification, an alternative indirect approach, referred to as the generalized sidelobe canceler (GSC), is adopted for implementing the LCAF problem. The GSC structure essentially decomposes the adaptive weight vector into constrained and unconstrained components. The unconstrained component can then be freely adjusted to meet any criterion since the constrained component will always ensure that the constraint equations are satisfied. The indirect implementation could attain the same performance as that using the direct constrained approach and possesses better numerical properties. Via computer simulation, the merits of the LC-IQRD-RLS algorithms over the conventional LC-RLS algorithm and its modified version are verified.

12.1 Introduction

The linearly constrained (LC) adaptive filtering (LCAF) technique is known to have many applications in areas such as minimum variance spectral analysis, time delay estimation, and antenna array signal processing [1–5]. More recently, these
constrained approaches have been applied to wireless communication systems for multiuser detection [6, 7]. Among the adaptive filtering algorithms, in most practical applications, the RLS algorithm has shown to offer better convergence rate and steady-state mean-squared error (MSE) over the least mean squares (LMS)-based algorithms. Unfortunately, the widespread acceptance of the RLS filter has been refrained due to numerical instability problems when it is implemented in limited-precision environments. Performance degradation is especially noticeable for the family of techniques collectively known as “fast” RLS filters [8–10]. To circumvent this problem, a well-known numerical stable RLS algorithm, which is called the QR-decomposition RLS (QRD-RLS) algorithm was proposed [8, 11–13]. It computes the QR decomposition (triangular orthogonalization) of the input data matrix using Givens rotation and then solves the LS weight vector by means of the back-substitution procedure.

The QRD-RLS algorithm can be implemented using the systolic array [14–17] with very large scale integration (VLSI) technology and DSP devices. However, in some practical applications, if the LS weight vector is desired in each iteration, back-substitution steps must be performed accordingly. Due to the fact that back-substitution is a costly operation to be performed in an array structure, the so-called inverse QRD-RLS (IQRD-RLS) algorithm proposed in [18] is preferred, for the LS weight vector can be computed without implementing back-substitution.

In this chapter, we first introduce the LC-RLS filtering algorithm based on an IQRD, where a direct form of recursively updating the constrained weight vector of LS solution is developed. The basic approach of the LC-IQRD-RLS algorithm is very similar to that discussed in [2]. That is, based on the Kalman gain of the conventional IQRD-RLS algorithm, the LC-IQRD-RLS algorithm can be developed where the unconstrained form of the weight vector and the \textit{a priori} estimation error can be avoided. In the IQRD-RLS algorithm, the parameters related to the Kalman gain are evaluated using the \textit{Givens rotations} (QR decomposition), which is quite different from the one discussed in [2] (using the fast RLS algorithm), yielding different development. Usually, the IQRD-RLS algorithm has better numerical accuracy than the “fast” RLS algorithm. Indeed, the numerical instability may cause the constraint drift problem [19] for the constrained approach based on the conventional fast RLS algorithm, named the constrained fast LS (CFLS) algorithm [2].

In this chapter, an alternative indirect approach, referred to as the generalized side-lobe canceler (GSC), is employed [6, 8, 20, 21] for various applications. The GSC structure essentially decomposes the adaptive weight vector into constrained and unconstrained components. The unconstrained component can then be freely adjusted to meet any criterion since the constrained component will always ensure that the constraint equations are satisfied. The GSC-based algorithm could attain the same performance as the direct constrained approach and possesses better numerical properties.