6
Gaussian Asymptotics for Power and Besov Norms

In this chapter we consider the alternatives $V_e$ under constraints determined by the power norms (2.71),

$$V_e = V(\kappa, \rho_e, R_e) = \{ v \in l^2 : |v|_{r,p} \geq \rho_e, |v|_{s,q} \leq R_e \};$$  \hspace{1cm} (6.1)

and by the Besov norms (2.72)

$$V_e = V(\tau, \rho_e, R_e) = \{ v \in l^2 : |v|_{r,p,h} \geq \rho_e, |v|_{s,q,t} \leq R_e \};$$  \hspace{1cm} (6.2)

(compare with (3.110), (3.122)); we denote $\kappa = (r, s, p, q)$, $\tau = (\kappa, h, t)$. We consider the general cases

$$-\infty < r, s < \infty, \quad 0 < p, h < \infty, \quad 0 < q, t \leq \infty,$$

for the rate asymptotics problem and add some additional constraints on $h, t$ for the sharp asymptotics problem. We assume the nonempty conditions from Corollary 3.15 and we suppose $\rho_e \to \infty$, $R_e \to \infty$.

These problems were studied above for the cases $\kappa \in \Xi_C$ (the set $\Xi_C$ is defined by (3.113) and corresponds to the asymptotics of classical type (3.114)), for $\kappa \in \Xi_T$ (the set $\Xi_T$ is defined by (3.116) and corresponds to the triviality; see Sections 3.4.6, 3.4.7), and for $\kappa \in \Xi_D$ (the set $\Xi_D$ is defined by (4.146) and corresponds to the asymptotics of degenerate type; see Section 4.4). Moreover the case $p \leq 2$, $q \geq p$ have been studied in Sections 4.3.3, 4.3.4 and we have obtained the asymptotics of Gaussian type.

Here we extend these results. We consider the case $\kappa \in \Xi_G$ where

$$\Xi_G = \{ \kappa : r \geq r^*_p, \kappa \notin \Xi_T, \kappa \notin \Xi_D \}. \hspace{1cm} (6.3)$$
We describe the regions $\Xi_{G_1}, \Xi_{G_2}$ of the principal types and the regions $\Xi_{G_3} - \Xi_{G_4}$ of frontier types of Gaussian asymptotics in the problems. Using the wavelet transform we translate the rate asymptotics to alternatives determined by Besov norms in the functional space (2.78) under a functional Gaussian model.

For the regions of the principal types, we show that the rate asymptotics for the Besov norms are the same as that for the power norms and do not depend on the additional parameters $t, h$. These provide the translations of the results to alternatives determined by Sobolev norms in the functional space.

6.1 Extreme Problems

We would like to apply Proposition 5.5 and to obtain the asymptotics (5.44) in the problems under consideration. The main point is the study of the extreme problem (5.40) with the sets $\Pi_\varepsilon$ specified for the problems of interest. In view of the considerations from Section 5.2.2, we can rewrite the extreme problem (5.40) in the form (5.64) where $H_l = H_{l,\varepsilon}$ and the functionals $G_l(\bar{\pi})$ are determined by the functionals $G_l(v)$, $l = 1, 2$, according to (5.53), (5.54). This has been discussed in Sections 5.2.2, 5.3 and we will combine the results.

First, let us consider the problem for power norms. If $q < \infty$, then the set $V_\varepsilon$ is of the form (5.45) with

$$G_1(v) = \sum_i i^r|v_i|^p, \quad H_{1,\varepsilon} = \rho_\varepsilon^p; \quad G_2(v) = \sum_i i^q|v_i|^q, \quad H_{2,\varepsilon} = R_\varepsilon^q.$$ 

If $q = \infty$, then

$$G_2(v) = \sup_i i^r|v_i|, \quad H_{2,\varepsilon} = R_\varepsilon.$$

Recall that the considerations from Section 5.2 lead to the extreme problem

$$u_\varepsilon^2 = u_\varepsilon^2(\kappa; \rho_\varepsilon, R_\varepsilon) = \inf_{i=1}^\infty \sum_i||\pi_i||^2$$

subject to, if $q < \infty$,

$$\pi_i \in \Pi, \quad \sum_{i=1}^\infty i^r E_{\pi_i} |t|^p \geq \rho_\varepsilon^p, \quad \sum_{j=1}^\infty i^q E_{\pi_i} |t|^q \leq R_\varepsilon^q$$

if $q = \infty$, then the last constraint in (6.5) is changed to $\sup_i i^s|\pi_i| \leq R_\varepsilon$. 

6. Gaussian Asymptotics for Power and Besov Norms