Boolean Dynamics with Random Couplings

Maximino Aldana
Susan Coppersmith
Leo P. Kadanoff

To Larry Sirovich, on the occasion of his 70th birthday.

ABSTRACT This paper reviews a class of generic dissipative dynamical systems called \( N-K \) models. In these models, the dynamics of \( N \) elements, defined as Boolean variables, develop step by step, clocked by a discrete time variable. Each of the \( N \) Boolean elements at a given time is given a value which depends upon \( K \) elements in the previous time step. We review the work of many authors on the behavior of the models, looking particularly at the structure and lengths of their cycles, the sizes of their basins of attraction, and the flow of information through the systems. In the limit of infinite \( N \), there is a phase transition between a chaotic and an ordered phase, with a critical phase in between. We argue that the behavior of this system depends significantly on the topology of the network connections. If the elements are placed upon a lattice with dimension \( d \), the system shows correlations related to the standard percolation or directed percolation phase transition on such a lattice. On the other hand, a very different behavior is seen in the Kauffman net in which all spins are equally likely to be coupled to a given spin. In this situation, coupling loops are mostly suppressed, and the behavior of the system is much more like that of a mean field theory. We also describe possible applications of the models to, for example, genetic networks, cell differentiation, evolution, democracy in social systems and neural networks.

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1 Introduction

In this review, we describe the dynamics of a set of $N$ variables, or elements, which each have two possible values (say 0 and 1). These elements interact with each other according to some given interaction rules, specified through a set of Boolean coupling functions that determine the variables at the next time-step, and thereby give the dynamics of the system. Such a discrete stepping of a set of Boolean variables, also known in general terms as a Boolean network, is of potential interest in several different fields, ranging from gene regulation and control, to modeling democracy and social organization, to understanding the behavior of glassy materials.

The models were originally studied primarily for their biological interest, specifically by Stuart Kauffman who introduced the so-called $N$-K model in the context of gene expression and fitness landscapes in 1969 (Kauffman [1969, 1974, 1995, 1993, 1990, 1984]). Since Kauffman’s original work, the scientific community has found a broad spectrum of applicability of these models. Specific biological problems studied include cell differentiation (Huang and Ingber [2000]), immune response (Kauffman and Weinberger [1989]), evolution (Bornholdt and Sneppen [1998]; Zawidzki [1998]; Bornholdt and Sneppen [2000]; Ito and Gunji [1994]), regulatory networks (Bornholdt and Rohlf [2000]) and neural networks (Wang, Pichler, and Ross [1990]; Derrida, Gardner, and Zippelius [1987]; Kürten [1988a]; Bornholdt and Rohlf [2000]). In the first two examples, the basic binary element might