Oscillatory Binary Fluid Convection in Finite Containers

Oriol Batiste
Edgar Knobloch

To Larry Sirovich, on the occasion of his 70th birthday.

ABSTRACT Linear and weakly nonlinear theory of overstable convection in large but bounded containers is reviewed and the results compared with detailed numerical simulations of binary fluid convection in a two-dimensional domain with realistic boundary conditions. For sufficiently negative separation ratios convection sets in as growing oscillations; the corresponding eigenfunctions take the form of 'chevrons' of either odd or even parity. These may bifurcate sub- or supercritically. Simulations of $^3\text{He}-^4\text{He}$ and water-ethanol mixtures show that the oscillations may equilibrate in finite amplitude chevron states, or that these states are unstable to blinking or repeated transient states. The results compare favorably with available experiments.

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1 Introduction

Binary fluid mixtures with a negative separation ratio exhibit a wide variety of behavior when heated from below. Particular attention has focused on the transition to various types of traveling waves with increasing Rayleigh number, hereafter $R$. The experimental situation is summarized by Sullivan and Ahlers [1988], Kolodner, Surko, and Williams [1989], Steinberg, Fineberg, Moses, and Rehberg [1989] and Kolodner [1993], and sample data are reproduced in fig. 1.1. These experiments have either been carried out in narrow gap annular containers, or in extended rectangular boxes. The two experimental arrangements differ in a fundamental way. In the former the system is periodic and consequently the initial instability can develop into a uniform pattern of traveling waves. This is no longer so when sidewalls are present: the presence of sidewalls destroys the translation invariance present in the annular (or unbounded) system, with the result that the finite system has only a left-right reflection symmetry. Consequently, the eigenfunctions of the latter system are either odd or even under left-right reflection, but are otherwise unconstrained by the symmetries [Dangelmayr and Knobloch, 1987, 1991; Dangelmayr, Knobloch, and Wegelin, 1991]. In contrast, in the annular (or unbounded) case the presence of translation invariance with periodic boundary conditions forces the eigenfunctions to be sinusoidal functions with a single wavenumber in the horizontal. Such eigenfunctions take the form of left- and right-traveling waves. In systems with up-down reflection symmetry the additional symmetry may also affect the eigenfunctions and constrain the dynamics.

The difference in symmetry between the bounded and unbounded systems is crucial, and is present regardless of the aspect ratio of the system. It suggests that while unbounded systems are best described in terms of amplitude equations for the amplitudes of left- and right-traveling waves, bounded systems should be described in terms of odd and even modes, cf. Landsberg and Knobloch [1996]. As shown by Batiste, Mercader, Net, and Knobloch [1999] these modes typically have a complex spatial structure. We summarize here the properties of these eigenfunctions for the parameter values used in experiments and relate them to two classes of weakly nonlinear theories developed for the onset of oscillatory instability in large aspect ratio domains. In particular we show that for large values of the aspect ratio $\Gamma$ the differences between the growth rates and frequencies of the first two modes that set in both scale as $\Gamma^{-2}$. This result supports the description of the system in terms of an interaction between the first even and odd modes [Landsberg and Knobloch, 1996]. Direct numerical simulations of the governing partial differential equations for both $^3$He-$^4$He and water-ethanol mixtures confirm the important role played by these pure parity modes, and shed light on the presence of two classes of dynamical behavior observed in the experiments, referred to as blinking states and repeated transients. We show that both of these states are fundamentally