Chapter 15

RESOURCE–CONSTRAINED PROJECT SCHEDULING WITH TIME WINDOWS

Recent developments and new applications

Klaus Neumann[1], Christoph Schwindt[2], Jürgen Zimmermann[2]

[1] *University of Karlsruhe, Germany;* [2] *Clausthal University of Technology, Germany;*

neumann@wior.uni-karlsruhe.de; {christoph.schwindt,juergen.zimmermann}@tu-clausthal.de

Abstract Recent results on resource–constrained project scheduling with time windows are reviewed. General temporal constraints (resulting from minimum and maximum time lags between project activities), several different types of scarce resources, and a large variety of time–based, financial, and resource–based objective functions are considered. Emphasis is placed on an order–based structural analysis of the feasible region of project scheduling problems and a classification and discussion of objective functions important to practice, which can be exploited for constructing efficient solution procedures. After those structural issues, methods for solving time–constrained project scheduling problems are proposed. Next, the resolution of conflicts for renewable, allocatable, synchronizing, changeover, and cumulative resources and thus the solving of corresponding resource–constrained project scheduling problems are studied. Finally, new applications of resource–constrained project scheduling are presented: factory pick–up of new cars and batch scheduling in process industries.

Keywords: Deterministic project scheduling, regular and nonregular objective functions, types of scarce resources, exact solution methods, customer–oriented factory pick–up, batch scheduling.

15.1 Introduction

Since the appearance of the earlier Handbook on Project Scheduling (cf. Węglarz 1999, Neumann and Zimmermann 1999), a large number of new results on resource–constrained project scheduling with schedule–dependent time windows have been published (see, e.g., Neumann et al 2000, Neumann and

Schwindt 2002, Neumann and Zimmermann 2002, Demeulemeester and Herroelen 2002, Neumann et al 2002b, 2003a,b, Schwindt and Trautmann 2003, Selle and Zimmermann 2003, Gentner et al 2004, Mellentien et al 2004, Neumann et al 2005, Schwindt 2005). These new results concern structural questions, a classification of time–based, financial and resource–based objective functions, different types of resources important in practice, efficient solution procedures, and new applications.

This chapter gives an overview of those new results on deterministic resource–constrained project scheduling with minimum and maximum time lags between activities. In Section 16.2, the basic project scheduling problem is formulated. Section 16.3 presents an order–based structural analysis of the feasible region of the basic project scheduling problem. Moreover, the classes of regular, convexifiable, locally regular, and locally concave objective functions are introduced and examples of such functions that are important to practice are given. Approaches to solving time–constrained project scheduling problems (i.e., without resource constraints) with the different types of objective functions introduced in Section 16.3 are discussed in Section 16.4. Section 16.5 deals with different types of resources important in practice and the resolution of so–called resource conflicts. In particular, renewable, cumulative (or storage), synchronizing, allocatable, and changeover resources are studied. In Section 15.6, we are concerned with new applications, such as factory pick–up of new cars and batch scheduling in process industries. Section 15.7 presents some conclusions and possible future research.

15.2 Basic project scheduling problem

We consider a project consisting of n *activities* $1, \ldots, n$. Let $p_i \in \mathbb{N}$ be the *duration* or *processing time* of activity i, which is assumed to be carried out without interruption. In addition, we introduce the *fictitious activities* 0 and $n+1$ representing the beginning and completion, respectively, of the project, where $p_0 = p_{n+1} = 0$. Then $V = \{0, 1, \ldots, n+1\}$ is the set of all activities.

Let $S_i \geq 0$ be the *start time* of activity $i \in V$, where $S_0 := 0$ (i.e., the project always begins at time zero). Then S_{n+1} represents the *project duration* or *makespan*. We assume that $S_{n+1} \leq \overline{d}$ where $\overline{d} \in \mathbb{N}$ is a prescribed maximum project duration or planning horizon. A sequence $S = (S_0, S_1, \ldots, S_{n+1})$ with $S_i \geq 0$ $(i \in V)$ and $S_0 = 0$ is called a *schedule*.

A *minimum time lag* $d_{ij}^{min} \in \mathbb{Z}_{\geq 0}$ or *maximum time lag* $d_{ij}^{max} \in \mathbb{Z}_{\geq 0}$ can be prescribed between the start of two different activities i and j, that is, $S_j - S_i \geq d_{ij}^{min}$ or $S_j - S_i \leq d_{ij}^{max}$, respectively. If $d_{ij}^{min} = p_i$, $S_j - S_i \geq d_{ij}^{min}$ represents a *precedence constraint*. To ensure that the project is terminated by time $\overline{d}$, we introduce the maximum time lag $d_{0,n+1}^{max} := \overline{d}$.