A GENERAL INDUCTIVE COMPLETION ALGORITHM
AND APPLICATION TO ABSTRACT DATA TYPES

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ABSTRACT: This paper states the connection between hierarchical construction
of equational specifications and completion of equational term rewriting
systems. A general inductive completion algorithm is given, which turns out to
be a well-suited tool to build up specifications by successive enrichments.
Moreover, the same algorithm allows verifying consistency of a specification or
proving theorems in its initial algebra without using explicit induction.

INTRODUCTION

In this paper, we address some general problems of equational logic
from the point of view of abstract data type specifications and term rewriting
systems. More precisely, we are dealing with three kinds of questions:

- How to prove theorems in the initial algebra of an equational theory,
  without using explicit induction?

- How to enrich a basic specification with new operators in a
  hierarchical way?

- How to prove correctness of some equational specification with
  respect to another one?

This work extends previous results obtained in the following framework:

Let F be a set of function symbols split into a set of constructors F₀
with at least two elements, and a set of defined function symbols F₁. We assume
that there is no relation between constructors and that the relations between
constructors and defined function symbols are expressed via a set of axioms A,
such that A provides a complete definition of operations of F₁. That implies

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that for any term, without variables, with at least one symbol of $F_1$, there exists an equivalent term modulo $A$ expressed with symbols of $F_0$ only. If in addition axioms of $A$ can be directed, giving a terminating and confluent term rewriting system $R$, the well-known Knuth and Bendix's completion procedure can be used to prove properties in the initial algebra of the equational theory $A$. After the pioneer work of Musser [MUS,80], the method has been clarified and improved by Goguen [GOO,80], Huet and Oppen [H&O,80], and Lankford [LAN,81]. In addition, Huet and Hullot [H&H,80] have shown how to modify the completion algorithm in order to take into account the existence of constructors, providing a powerful technique. These works have given rise to various implementations: the systems AFFIRM, OBJ, FORMEL, REVE. Some of them accept associative commutative symbols in $F_1$, using for instance in FORMEL, the Peterson and Stickel's completion algorithm [P&S,81].

In order to allow relations between constructors, Remy [REM,82] introduced the concept of structured specification, in which equations on constructors can be directed into a confluent and noetherian term rewriting system.

On the other hand, Jouannaud [JOU,83] developed a general framework to decide equality in equational theories where some axioms (like commutativity) cannot be directed without losing the termination property. An equational term rewriting system is then composed of a set of equations $E$ and a set of rewrite rules $R$; a suitable Church-Rosser property allows deciding $(R \cup E)$-equality of two terms by using rewritings only. In [J&K,84] and [KIR,83], we proved a general completion algorithm, roughly based on the same principles as the Knuth and Bendix's one, which allows completing an equational term rewriting system into another one which has the Church-Rosser property and defines the same congruence on terms. Our aim is to adapt this completion algorithm in order to deal with the problems of consistency or proofs in initial algebra, but for the wider class of equational theories which admit an equational term rewriting system. Moreover this inductive completion algorithm turns out to be a suitable tool for proofs in hierarchical specifications.