A DECISION METHOD FOR LINEAR TEMPORAL LOGIC

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Abstract

In this paper we define a new decision method for propositional temporal logic of programs. Temporal logic appears to be an appropriate tool to prove some properties of programs such as invariance or eventually because in this logic we define operators that enable us to represent properties that are valid during all the development of the program or over some parts of the program.

The decision method that we define is an extension of classical resolution to temporal operators.

We give an example of a mutual exclusion problem and we show, using resolution method, that it verifies a liveness property.

Introduction

In this paper we define a new decision method for propositional linear temporal logic of programs such as defines in [BM], [GPSS], [MZ].

Temporal logic appears to be an appropriate tool to prove some properties of programs like invariance, eventuality and precedence because in this logic we define operators that enable us to represent formally properties that are valid during all the development of the program, over some parts of the program or properties stating that a certain event always precedes another.

The decision method that we define is an extension of classical resolution [RJ], to temporal operators; to obtain it we will follow
the same way as proposed in [FC]. First, we define a conjunctive normal form for the formulas of temporal logic, section 1; after that we define transformation rules to obtain comparable formulas, section 2, and finally we define, in section 3, resolution rules for modal formulas in a way analogous of classical resolution and we prove that the rule system is complete, section 4.

In section 5, we give a simple example of a mutual exclusion problem and we show, using resolution method, that it verifies a liveness property.

We define formulas as words in a finite alphabet. A modal formula is either a literal (propositional variable or negation of a propositional variable) or has the form: \((A \& B), (A \lor B), \neg A, \forall A, (\square A)\) (\(\square A\) means necessary \(A\)), \(\diamond A\) (\(\diamond A\) means next \(A\)), where \(A\) and \(B\) are modal formulas.

A modal system is a set of modal formulas.

A \(w\)-structure is a pair \(<W,\preceq>\), where \(W\) is a sequence \(w_0, w_1, w_2, \ldots\) the states, and \(w_i \preceq w_j\) iff \(i \leq j\). And an assignment \(V\) for \(<W,\preceq>\) assigns to each propositional variable \(p\) a subset \(V(p)\) of \(W\).

Given an assignment \(V\) for \(<W,\preceq>\), we define \(V(A,w_i) \in \{T,F\}\), where \(A\) is a formula and \(w_i \in W\), according to the customary inductive definition; in particular:

\[
V(\diamond A, w_i) = T \ \text{iff} \ V(A, w_{i+1}) = T, \ \text{and} \\
V(\square A, w_i) = T \ \text{iff} \ V(A, w_j) = T \ \text{for every} \ w_j \in W \ \text{such that} \ w_i \preceq w_j
\]

We say that \(A\) is valid (unsatisfiable) in \(W\) if \(V(A, w_i) = T(F)\) for every assignment \(V\) for \(W\) and every \(w_i \in W\).

We consider the following abbreviations:

\[
\diamond A = \sim \square \sim A \ (\diamond A \text{ means possibly } A) \\
\diamond^n A = A \lor \diamond A \lor \ldots \lor \diamond^{n-1} A \ \text{notes the string } \diamond \ldots \diamond A \ \text{n-1 times} \\
\square^n A = A \land \diamond A \land \ldots \land \diamond^{n-1} A
\]

The axiomatic system for propositional linear temporal logic is obtained by adding to the usual formalisation of propositional calculus the following axioms and inference rule: