18 Scale, Scaling and Multifractals in Geophysics: Twenty Years on

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Abstract. We consider three developments in high number of degrees of freedom approaches to nonlinear geophysics: a) the transition from fractal geometry to multifractal processes, b) the self-organized critical (SOC) generation of extremes via multifractal phase transitions c) the generalization from isotropic scale invariance (self-similar fractals, multifractals) to (anisotropic) generalized scale invariance. We argue that these innovations are generally necessary for geophysical applications. We illustrate these ideas with data analyses from both the atmosphere and the earth’s surface, as well as with multifractal simulations.

1 Introduction: Which Chaos in Geophysics?

During the 1960’s, 70’s and 80’s theoretical developments in geophysics, physics, and mathematics spawned four related non-linear paradigms. The first, deterministic chaos, was centred around the discovery [Lorenz, 1963], [Ruelle and Takens, 1971] that systems with as few as three degrees of freedom could have random-like “chaotic” behaviour. The second, fractal geometry - proposed that many natural systems could be modeled as (stochastic, scale invariant) fractal sets [Mandelbrot, 1967]. The third, “self-organized criticality” (SOC) [Bak, et. al., 1987] proposed that extreme events could be the result of seemingly simple generic avalanche-like processes. As discussed below SOC turns out to have close relationships with non classical critical phase transitions, which help to establish links to the fourth scaling tool, the “renormalization group”. The latter has been extremely helpful in clarifying classical phase transitions [Wilson, 1971] and in understanding the dynamical nature of scaling. By the early 1980’s further developments had made the first two quite practical. In particular, the discovery of universality in chaos by [Feigenbaum, 1978] had emphasized the generic features of chaotic models rendering them more applicable, and the revolution in computer graphics had made fractals – including “strange” chaotic attractors – palpable.

This short expose gives an overview of certain subsequent developments covering roughly the twenty years celebrated by the conference. While deterministic chaos is
essentially a low degrees of freedom paradigm, fractals and SOC are both high number of degrees of freedom frameworks and could thus be called “stochastic chaos” since they involve infinite dimensional probability spaces [Lovejoy and Schertzer, 1998a]. While the question of whether deterministic or stochastic chaos is more geophysically relevant continues to be debated, here we focus on the latter (in particular Schertzer et al., 2002 who showed that a finite correlation dimension does not discriminate between deterministic and stochastic chaos). We outline three key developments which allow the fractal and SOC paradigms to be widely applicable in geosciences: a) the transition from scale invariant sets (fractals) to scale invariant fields (multifractals), b) the recognition of the link between extreme events, heavy tailed (algebraic) probabilities (SOC) and space-time scaling, c) the generalization of scale invariance from isotropic (self-similar) systems to very general anisotropic ones (self-affine and beyond) within the framework of Generalized Scale Invariance (GSI). Rather than attempt a systematic survey, we illustrate our discussion using results from key fields in solid earth and atmospheric geophysics: the earth’s topography and clouds and rain. This choice is motivated by both the fundamental significance of the fields and for the availability of relevant high quality data.

2 The Link between Descriptions and Models

It is an old truism that one cannot make a measurement without first having a theory of what is to be measured. This is well illustrated in nonlinear geophysics where theoretical developments are not only necessary for making more sophisticated theories and better applications: they are necessary simply in order to quantitatively describe geofields. We illustrate this statement with two significant examples. The first is the long debate starting in the 1980’s about what was the (supposedly unique) fractal dimension of the earth’s topography. If the topography could be adequately modeled as a geometrical fractal set, then many different techniques (including spectral analysis) could be used to estimate its unique dimension $D$. Unfortunately, different techniques applied to different data commonly gave different values of $D$ (see the review in Klinkenberg and Goodchild, 1992]). Consequently by the end of the 1990’s the mainstream surface geomorphology community had “moved on”, relegating fractals to narrow ranges of scale and to very technical applications. This near abandonment of scaling occurred in spite of the fact that entire fields of research such as surface hydrology (see e.g. the excellent review [Rodriguez-Iturbe and Rinaldo, 1997]) are riddled with scaling laws which virtually require the topography to respect some form of scaling. At the same time, due to their random singularities, multifractals have such strong variability that they violate many conventional geostatistical assumptions so that normal multifractal variability can easily be misinterpreted in terms of spurious scale breaks, spurious nonstationarity etc. The loss of interest in scaling was encouraged by the extensive use of (low variability) fractional Brownian motion (fBm) models of topography. As argued in [Gagnon, et al., 2006], the topography in fact has excellent multiscaleing (multifractal) properties (see Fig. 18.2, 18.4, 18.7) – but an infinite hierarchy of fractal dimensions; this requires new analysis techniques. Consequently