In this chapter we will consider two recent applications of concepts and techniques from algebraic geometry in areas of computer science. First, continuing a theme introduced in several examples in Chapter 1, we will develop a systematic approach that uses algebraic varieties to describe the space of possible configurations of mechanical linkages such as robot “arms.” We will use this approach to solve the forward and inverse kinematic problems of robotics for certain types of robots.

Second, we will apply the algorithms developed in earlier chapters to the study of automatic geometric theorem proving, an area that has been of interest to researchers in artificial intelligence. When the hypotheses of a geometric theorem can be expressed as polynomial equations relating the cartesian coordinates of points in the Euclidean plane, the geometrical propositions deducible from the hypotheses will include all the statements that can be expressed as polynomials in the ideal generated by the hypotheses.

§1 Geometric Description of Robots

To treat the space of configurations of a robot geometrically, we need to make some simplifying assumptions about the components of our robots and their mechanical properties. We will not try to address many important issues in the engineering of actual robots (such as what types of motors and mechanical linkages would be used to achieve what motions, and how those motions would be controlled). Thus, we will restrict ourselves to highly idealized robots. However, within this framework, we will be able to indicate the types of problems that actually arise in robot motion description and planning.

We will always consider robots constructed from rigid links or segments, connected by joints of various types. For simplicity, we will consider only robots in which the segments are connected in series, as in a human limb. One end of our robot “arm” will usually be fixed in position. At the other end will be the “hand” or “effector,” which will sometimes be considered as a final segment of the robot. In actual robots, this “hand” might be provided with mechanisms for grasping objects or with tools for performing some task. Thus, one of the major goals is to be able to describe and specify the position and orientation of the “hand.”
Since the segments of our robots are rigid, the possible motions of the entire robot assembly are determined by the motions of the joints. Many actual robots are constructed using

- planar revolute joints, and
- prismatic joints.

A planar revolute joint permits a rotation of one segment relative to another. We will assume that both of the segments in question lie in one plane and all motions of the joint will leave the two segments in that plane. (This is the same as saying that the axis of rotation is perpendicular to the plane in question.)

A prismatic joint permits one segment of a robot to move by sliding or translation along an axis. The following sketch shows a schematic view of a prismatic joint between two segments of a robot lying in a plane. Such a joint permits translational motion along a line in the plane.

If there are several joints in a robot, we will assume for simplicity that the joints all lie in the same plane, that the axes of rotation of all revolute joints are perpendicular to that plane, and, in addition, that the translation axes for the prismatic joints all lie