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MOEA Theory and Issues

> He who loves practice without theory is like the sailor who boards ship without a rudder and compass and never knows where he may cast.
>
> Leonardo da Vinci

6.1 Introduction

Many MOEA development efforts acknowledge various facets of underlying MOEA theory, but make limited contributions when simply citing relevant issues raised by others. Some authors, however, exhibit significant theoretical detail. Their work provides basic MOEA models and associated theories. Table 6.1 lists contemporary efforts reflecting MOEA theory development. In essence, a MOEA is searching for optimal elements in a partially ordered set or in the Pareto optimal set. Thus, the concept of convergence to P_{true} and PF_{true} is integral to the MOEA search process.

As observed, MOEA theory noticeably lags behind applications, at least in terms of published papers. This is even clearer when noting few of these categorized papers (see Chapter 2) *concentrate* on MOEA theoretical concerns. Others discuss some MOEA theory but do so only as regarding various parameters of their respective approaches. This quantitative lack of theory is not necessarily bad but indicates further theoretical development is necessary to (possibly) increase the effectiveness and efficiency of existing MOEAs. The rest of this chapter is organized as follows. Section 6.2 presents various MOEA theoretical definitions, theorems, and corollaries. Appropriate mathematical definitions are provided to support understanding of theorems and corollaries. Also, note that certain theorems require specific MOEA structures in order to prove convergence. Section 6.3 discusses MOEA issues as related to contemporary theoretical results including fitness functions, fitness landscapes, Pareto ranking, niching, mating restriction, running time analysis and stability. The chapter concludes with some research ideas and discussion questions.

Table 6.1: MOEA Theory

Researcher(s)	Paper Focus
Fonseca and Fleming [510]	MOEA mathematical formulations
Rudolph [1396], Rudolph & Agapie [1401]	MOEA convergence
Veldhuizen and Lamont [1627]	MOEA convergence and Pareto terminology
Veldhuizen and Lamont [1630]	MOEA benchmark test problems
Hanne [643]	MOEA convergence and Pareto terminology
Laumanns et al. [959], Knowles and Corne [877]	Archiving techniques
Deb & Meyarivan [371], Deb et al. [375]	Constrained test problems
Rudolph [1395, 1397]	MOEA search under partially ordered sets
Ehrgott [427]	Analysis of the computational complexity of multiobjective combinatorial optimization problems
Rudolph [1399]	Limit theory for EAs under partially ordered fitness sets
Laumanns et al. [962, 960]	Running time analysis
Laumanns et al. [958]	Mutation control
Hanne [644]	Convergence to the Pareto optimal set
Aguirre and Tanaka [16], Knowles and Corne [875]	Fitness landscapes

6.2 Pareto-Related Theoretical Contributions

Pareto-based theorems and definitions have been developed to support research objectives and other theoretical results as reflected in Table 6.1. Many MOEAs assume each generational population contains Pareto optimal solutions (with respect to that population). For example, Theorem 1 substantiates this assumption. As the MOEA literature offers little guidance concerning possible Pareto front cardinality and dimensionality, Theorems 4 and 5 provide an upper bound. Thus, these and other theoretical Pareto contributions as referenced further bounding both problem and algorithm domains. Appropriate ones are presented here for coherence. Some theorems are presented without proof, but such proofs can be found in the associated references. Others have been reworded in order to reflect the nomenclature used in this book. A limited number of symbols have also been employed for ease of initial understanding. But, first the associated definitions of partially ordered sets are discussed as related to individual fitness function values.

6.2.1 Partially Ordered Sets

Partially ordered sets are an integral aspect of moving a population towards the Pareto front using dominance. The underlying concept of a mathematical relation provides the basic foundation for studying partially ordered sets over MOEA fitness functions. Note that optimality is assumed to reflect a minimization MOP in the continuing discussion.

Definition 35 (Relation) : *Let x, y and z be in X, some set satisfying a binary relation R such that xRy satisfies one or more of the following properties:*