Sensitivity Analysis in Monte Carlo Simulation of Stochastic Activity Networks

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Summary. Stochastic activity networks (SANs) such as those arising in Project Evaluation Review Technique (PERT) and Critical Path Method (CPM) are an important classical set of models in operations research. We focus on sensitivity analysis for stochastic activity networks when Monte Carlo simulation is employed. After a brief aside reminiscing on Saul's influence on the author's career and on the simulation community, we review previous research for sensitivity analysis of simulated SANs, give a brief summary overview of the main approaches in stochastic gradient estimation, derive estimators using techniques not previously applied to this setting, and address some new problems not previously considered. We conclude with some thoughts on future research directions.

Key words: Stochastic activity networks; PERT; CPM; project management; Monte Carlo simulation; sensitivity analysis; derivative estimation; perturbation analysis; likelihood ratio/score function method; weak derivatives.

1 Introduction

In the vast toolkit of operations research (OR), two of the most useful methods/models without a doubt are simulation and networks. Numerous surveys of practitioners consistently place these in the top 10 in terms of applicability to real-world problems and solutions. Furthermore, there is a large industry of supporting software for both domains. Thus, almost all degree-granting programs in operations research offer standalone courses covering these two topics. At the University of Maryland's Robert H. Smith School of Business, these two courses at the Ph.D. level have course numbers BMGT831 and BMGT835. They form half of the core of the methodological base that all OR Ph.D. students take, along with BMGT834 Stochastic Models and BMGT830 Linear programming, which Saul Gass taught for much of his career at Maryland,

using his own textbook first published in 1958, and translated into numerous other languages, including Polish, Russian, and Spanish. Reflecting back on my own academic career, the first presentation I ever gave at a technical conference was at the 1988 ORSA/TIMS Spring National Meeting in Washington, D.C., in which Saul was the General Chair. When the INFORMS Spring Meeting returned to Washington, D.C. in 1996, Saul was again on the advisory board and delivered the plenary address, and this time, through my connection with him, I served on the Program Committee, in charge of contributed papers.

Saul's contributions to linear programming and his involvement in ORSA and then INFORMS are of course well known, but what may not be as well known are his contributions to the simulation community and simulation research. Saul was heavily involved with the Winter Simulation Conference — the premier annual meeting of stochastic discrete-event simulation researchers, practitioners, and software vendors — during what could be called the formative and critical years of the conference. Saul served as the ORSA representative on the Board of Directors in the early 1980s. During these years, "he contributed much insight into the operation of conferences and added prestige" [21], which clearly helped launch these meetings on the path to success. In addition, Saul has also contributed to simulation research in model evaluation and validation through "(i) the development of a general methodology for model evaluation and validation [15, 18], and (ii) the development of specific validation techniques that used quantitative approaches [16, 17]" [21]. Arjang Assad's article in this volume details numerous additional instances of Saul's involvement with simulation during his early career.

The networks studied in this paper are a well-known class of models in operations research called *stochastic activity networks* (SANs), which include the popular Project Evaluation Review Technique (PERT) and Critical Path Method (CPM). A nice introduction to PERT can be found in the encyclopedia entry written by Arjang Assad and Bruce Golden [2], two long-time colleagues of Saul hired by him in the 1970s while he was chairman of the Management Science & Statistics department at the University of Maryland, and co-authors of other chapters in this volume. Such models are commonly used in resource-constrained project scheduling; see [6] for a review and classification of existing literature and models in this field. In SANs, the most important measures of performance are the completion time and the arc (activity) criticalities. Often just as important as the performance measures themselves are the sensitivities of the performance measures with respect to parameters of the network. These issues are described in the state-of-the-art review [8]. When the networks become complex enough, Monte Carlo simulation is frequently used to estimate performance. However, simulation can be very expensive and time consuming, so finding ways to reduce the computational burden are important. Efficient estimation schemes for estimating arc criticalities and sensitivity curves via simulation are derived in [4, 5].