Recovering Circles and Spheres from Point Data

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Summary. Methods for fitting circles and spheres to point sets are discussed. LADAR (LAser Detection And Ranging) scanners are capable of generating "point clouds" containing the (x, y, z) coordinates of up to several millions of points reflecting the laser signals. In particular, coordinates collected off objects such as spheres may then be used to model these objects by fitting procedures. Fitting amounts to minimizing what is called here a "gauge function," which quantifies the quality of a particular fit. This work analyzes and experimentally examines the impact of the choice of three such gauge functions. One of the resulting methods, termed here as "algebraic" fitting, formulates the minimization problem as a regression. The second, referred to as "geometric" fitting, minimizes the sum of squares of the Euclidean distances of the data points from the tentative sphere. This method, based on orthogonal distance minimization, is most highly regarded and widely used. The third method represents a novel way of fitting. It is based on the directions in which the individual data points have been acquired.

Key words: algebraic fitting; circles; coordinate search; directional fitting; geometric fitting; LADAR; optimization; quasi-Newton; registration; spheres.

1 Introduction

In 1997, an article by Rorres and Romano [19] addressing an archaeological problem caught the attention of Saul Gass. In the Greek city of Corinth, the circular starting line for an ancient (circa 550 B.C.) track for foot races had been found. The other end of such a racetrack had to be marked by a turning pole [18]. The circular starting line was chosen, presumably, to equalize the distances between starting positions and the turning pole at the far end. The location of the turning pole could thus be inferred as the center of the circle passing through the starting line. This is precisely what Rorres and Romano did: they surveyed points on the starting line and fit a circle to these points.

Saul was intrigued by this problem of recovering a circle from a set of points as he had encountered circle and sphere fitting problems earlier in con-

nection with Coordinate Measuring Machines (CMMs), which are crucial to today's precision manufacturing. Their metrology is a major concern at the National Institute of Standards and Technology (NIST) [17], with which he has been associated for nearly three decades as an academic advisor. The computer-guided probe of a CMM touches an object and records to a high level of accuracy the (x, y, z) coordinates of the point on the object where contact was made. A collection of such measured points then permits an accurate assessment of the shape of the object, say, the roundness of a sphere, or its dimensions, say, the radius of a sphere. Captivated by the obvious parallels between such geometric problems and those encountered in classical Operations Research [5, 6], he explored the use of linear programming in this context [9]. He also dug deeper into the racetrack problem, using several different methods for fitting the data and, in particular, compared the widths of "annuli," the areas between two concentric circles containing all data points. He was able to achieve tighter results than those reported by Rorres and Romano. A beautifully crafted unpublished manuscript [10] summarizes this work. It is also telling that his emphasis was not so much on "how" to compute, but rather on "what" to compute, in other words, the task of "modeling" so as to capture a particular aspect of reality.

Our work aspires to follow his example. It was prompted by the rapid growth of 3D imaging technology and its applications, and the corresponding need for metrological analysis. 3D imaging systems include laser scanners and optical range cameras. The former category covers LADARs (LAser Detection And Ranging) or laser radars. Similarly to a CMM, a LADAR also determines 3D coordinates of points on an object, but does so by sending out a laser signal and analyzing its reflection back to the instrument as indicated in Figure 1. Also, a LADAR is typically a scanning device that can obtain millions of measurements in a very short time, resulting in large "point clouds" of possibly millions of data points. Applications include the monitoring of construction sites, the development of "as built" models of existing structures, mapping, visualization of hidden objects, guiding of unmanned vehicles, just to mention a few.

1.1 LADAR Technology; Point Clouds

The metrology of LADARs is a current research issue at NIST. This work also supports the development of standard test protocols for the performance evaluation of LADARs. Figure 2 shows the indoor, artifact-based LADAR facility at NIST. Outdoor test facilities are planned for instruments with ranges of 100 m and above.

Figure 3 presents a LADAR visualization of a rubble pile. The casual reader may see this picture as just what a photographic camera might produce. But this would miss the point that, once the point cloud has been captured in 3D, it can be displayed as seen from different view points. For instance, the