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What Is a Proof and Why?

The proof of the pudding is in the eating. —Miguel Cervantes

In mathematics there are no true controversies. —Carl Friedrich Gauss

Logic is the art of going wrong with confidence. —Anonymous

To test man, the proofs shift. —Robert Browning

Newton was a most fortunate man because there is just one universe and Newton had discovered its laws. —Pierre-Simon Laplace

The chief aim of all investigations of the external world should be to discover the rational order and harmony which has been imposed on it by God and which He revealed to us in the language of mathematics. —Johannes Kepler

A terrier may not be able to define a rat, but a terrier knows a rat when he sees it. —A. E. Housman

1.1 What Is a Mathematician?

A well-meaning mother was once heard telling her child that a mathematician is someone who does “scientific arithmetic.” Others think that a mathematician is someone who spends all day hacking away at a computer.

Neither of these contentions is incorrect, but they do not begin to penetrate all that a mathematician really is. Paraphrasing mathematician/linguist Keith Devlin, we note that a mathematician is someone who:
1. What Is a Proof and Why?

- observes and interprets phenomena
- analyzes scientific events and information
- formulates concepts
- generalizes concepts
- performs inductive reasoning
- performs analogical reasoning
- engages in trial and error (and evaluation)
- models ideas and phenomena
- formulates problems
- abstracts from problems
- solves problems
- uses computation to draw analytical conclusions
- makes deductions
- makes guesses
- proves theorems

And even this list is incomplete. A mathematician has to be a master of critical thinking, analysis, and deductive logic. These skills are robust, and can be applied in a large variety of situations—and in many different disciplines. Today, mathematical skills are being put to good use in medicine, physics, law, commerce, Internet design, engineering, chemistry, biological science, social science, anthropology, genetics, warfare, cryptography, plastic surgery, security analysis, data manipulation, computer science, and in many other disciplines and endeavors as well.

One of the astonishing and dramatic new uses of mathematics that has come about in the past twenty years is in finance. The work of Fischer Black of Harvard and Myron