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# Weighted Back-projection Methods

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## *1. INTRODUCTION*

Traditionally, 3D reconstruction methods have been classified into two major groups, *Fourier reconstruction methods* and *direct methods* (e.g. Crowther *et al.*, 1970; Gilbert, 1972). Fourier methods are defined as algorithms that restore the Fourier transform of the object from the Fourier transforms of their projections and then obtain the real-space distribution of the object by inverse Fourier transformation. Included in this group are also equivalent reconstruction schemes that use expansions of object and projections into orthogonal function systems (e.g. Cormack, 1963, 1964; Smith *et al.*, 1973; Chapter 9 of this volume). In contrast, direct methods are defined as those that carry out all calculations in real space. These include the convolution back-projection algorithms (Bracewell and Riddle, 1967; Gilbert, 1972; Ramachandran and Lakshminarayanan, 1971) and iterative algorithms (Gordon *et al.*, 1970; Colsher, 1977). Weighted back-projection methods are difficult to classify in this scheme, since they are equivalent to convolution back-projection algorithms, but work on the real-space data as well as the Fourier transform data of either the object or the projections. Both convolution back-projection and weighted back-projection algorithms are based on the same theory as Fourier reconstruction methods, whereas iterative methods normally do not take into account the Fourier relationships between object transform and projection transforms. Thus it seems justified to classify the reconstruction algorithms into three groups: *Fourier reconstruction methods*, *modified back-projection methods* and *iterative direct space methods*, where the second group includes convolution back-projection as well as weighted back-projection methods.

Each reconstruction algorithm requires a set of projections of the object, recorded under different projecting directions. While the choice of the data collection geometry is determined by the properties of the specimen, e.g. by its radiation sensitivity, by the degree of variation among particles in the preparation and by orientational preferences of the particles, this geometry in turn determines which algorithm is most efficient in calculating the 3D structure.

As explained in more detail in the introductory chapter by Frank, four kinds of data collection schemes are used in electron microscopy: single-axis tilting, conical tilting, random-conical tilt and general random tilt. For the collection of a single-axis tilt series, the specimen is tilted in the microscope in a range of typically –60° to +60° in small increments, e.g. 1–5°, and an image of the same particle is recorded for each specimen position. For a conical tilt series, the specimen is tilted by one fixed angle in the range of 45–60° and then rotated within this plane by small angular increments. Again an image of the same particle is recorded for each specimen position. Both data collection schemes are mainly used for preparations that are radiation resistant or contain particles that individually have different shapes, i.e. averaging over different particles is either not required or not