2. Geometry of reflector antennas

In this chapter we deal with a description of the geometry of a paraboloid of revolution. We shall collect the geometrical relationships, which we need for the description of the electromagnetic radiation characteristics of the paraboloidal reflector antenna. Because many radio telescopes and communication antennas actually employ the Cassegrain or Gregorian layout, we include the formulas for those systems too. These are dual reflector systems, where a relatively small secondary reflector, placed near the focus of the primary paraboloid concentrates the received radiation in a secondary focus, located near or behind the vertex of the primary reflector. The Cassegrain employs a hyperboloidal secondary, while the Gregorian system uses an elliptical secondary reflector. One of the foci of these dual-focus conic sections coincides with the focus of the paraboloid, while the other provides the secondary focus at a convenient location. The great advantage from an operational viewpoint is the possibility to locate bulky receiving equipment behind the primary reflector. As we shall see later, there are also significant electro-magnetic advantages of the dual-reflector varieties.

In the second half of this chapter we discuss the aspects of imperfections in the geometry of the reflector system. In particular, the cases of "defocus" are described, where the detector element is displaced from the true focal point. This leads to the so-called aberrations in the optical system. We treat those by calculating the path-length differences over the reflector. These are then introduced as phase errors in the electromagnetic analysis of the antenna in a later chapter.

2.1. Geometrical relations of the dual reflector system

The geometrical definition of a parabola is illustrated in Fig. 2.1, where we limit ourselves to the two-dimensional case. Consider a coordinate system \((x, z)\) and choose a point \(F\) on the \(z\) axis, at a distance \(f\) from the origin. Draw a line perpendicular to the \(x\)-axis through \(F\) and choose a point \(Q\) at coordinate \((x, z)\). The definition of the parabola is the locus of points \(P\), where the sum of the distance from point \(P\) to \(F\) and the distance from \(P\) to \(Q\), with \(PQ\) parallel to the \(z\)-axis, is constant. Now assign \(Q\) to \(F\). Then \(P\) moves to the origin \(O\) and we have \(\text{"QP"} + \text{"PF"} = \text{"FO"} + \text{"OF"} = 2f\). For arbitrary value \(x\) of \(Q\) on the line through \(F\) we have

\[
PQ = f - z_p \quad \text{and} \quad FP^2 = FQ^2 + PQ^2 = x_p^2 + (f - z_p)^2,
\]

where \(z_p\) is the \(z\)-coordinate of \(P\).
The definition of the parabola now results in the following equation

\[ FP + PQ = \sqrt{x_p^2 + (f - z_p)^2} + f - z_p = 2f, \]

from which it is easy to derive the defining equation for the parabola in cartesian coordinates as

\[ x^2 = 4fz. \]

The definition in spherical coordinates \((\rho, \psi)\) is even simpler:

\[ FP + PQ = \rho + \rho \cos \psi = 2f, \]

from which follows

\[ \rho = 2f / (1 + \cos \psi). \]

From the geometrical definition it is easy to see the physical significance of the parabola. Let a bundle of light rays travel parallel to the z-axis. Upon reflection at the parabola, each ray will arrive at point F along equal path length and the intensities of the rays will be added there. The point F is the focus of the parabolic mirror. In terms of wave fronts we can say that a plane wave, traveling along the z-axis is transferred upon reflection at the parabola into a spherical wave converging towards the focal point F and adding the field contributions in phase. Conversely, a source of spherical waves in F will cause a plane wave traveling along the z-axis after reflection at the parabola.

In the Cassegrain antenna a hyperbolic secondary reflector is used to transfer the spherical wave front traveling from the parabola into another spherical wave front towards a secondary focus. The definition of a hyperbola is the locus of points, where the difference between the distances to two fixed points is constant. From the geometry of Figure 2.1 we have \(PF' - PF = \) constant. If we put \(P \) on the z-axis, it is immediately clear that the constant = \(2a\). In the general case of arbitrary \(P \) we have