3. Electromagnetic theory of the reflector antenna

In this chapter we develop the general theory of the reflector antenna with a primary source of radiation (the "feed") in the focal point. We will not attempt to present a full rigorous theory of the radiation characteristics over the entire space around the antenna. Rather our primary interest is to find a representation of the radiation pattern in a limited solid angle around the boresight axis with a sufficient accuracy to reliably derive the important antenna parameters, as gain, sidelobe level, beam efficiency, etc. We include the influence of the finite distance of the field-point from the antenna. Thus we describe the radiation function both in the Fraunhofer (farfield) region at infinite distance, as well as in the Fresnel region, where the distance to the field point is finite.

We depart from Maxwell's equations and derive the wave equation which governs the fields radiated from a distribution of electric currents and charges in a volume of space bounded by a closed surface. We apply this to the case of a reflector antenna fed by a plane wave source in the focal point. The field from the feed induces surface currents on the reflector surface, which in turn are the sources of electro-magnetic radiation. The field strength in a point P of space, outside the antenna, is found from an integration of the currents on the illuminated reflector surface. Some approximations are introduced in this procedure. These lead to a representation of the radiated field as an integration of the field projected from the surface onto the aperture plane of the reflector; the so-called Kirchhoff-Helmholtz aperture integration. This approximation is sufficiently accurate for most applications in the area of large antennas, reckoned in terms of the wavelength used. The aperture integration is applied to a circular aperture and we obtain the radiation characteristics for field points both in the Fraunhofer and Fresnel regions of the antenna. The detailed discussion of these, including the effects of aberrations is reserved for the following chapter. We conclude this chapter with a short discussion of the interesting fact that the aperture field and the farfield are related by a Fourier Transformation. From a reciprocity relation it can also be derived that the spatial field distribution in the focal plane is identical to the field distribution in the farfield.

3.1. Basic theory - Maxwell's equations

The description of any electromagnetic phenomenon must of course start with obeying the electromagnetic field equations of Maxwell. For a homogeneous, isotropic and linear medium, outside of the sources, these take the form
Here \( \mathbf{H} \) and \( \mathbf{E} \) denote the magnetic and electric field strength, respectively, \( \mathbf{j} \) is the electric current density, \( \rho \) the electric charge density, while \( \epsilon \) and \( \mu \) are the permittivity and permeability of the medium. Upon introduction of the vector potential \( \mathbf{A} \), defined by \( \mathbf{H} = \frac{1}{\mu} \nabla \times \mathbf{A} \), one can easily show that \( \mathbf{A} \) can be made subject to the following wave equation

\[
\Delta \mathbf{A} - \frac{1}{c^2} \frac{\partial^2 \mathbf{A}}{\partial t^2} = -\mu \mathbf{j}.
\]  

(3.2)

As usual, we assume harmonic time dependence of the fields, written as \( \exp(-i \omega t) \), which reduces the time dependent members of Eq. (3.1) to

\[
\nabla \times \mathbf{H} + i \omega \epsilon \mathbf{E} = \mathbf{j} \quad \text{and} \quad \nabla \times \mathbf{E} - i \omega \mu \mathbf{H} = 0.
\]

(3.3)

Introducing the wave number \( k \) with \( k^2 = \omega^2/\mu \), the wave equation now becomes

\[
\Delta \mathbf{A} + k^2 \mathbf{A} = -\mu \mathbf{j}.
\]

(3.4)

A solution of Eq. (3.4) is

\[
\mathbf{A} = \frac{\mu}{4\pi} \int_V \mathbf{j} \Psi \, dV,
\]

(3.5)

where \( \Psi = \exp(i k r)/r \) is the so-called Green function and the integration is carried out over the volume containing all current sources. Let us now divide space into a volume \( V_1 \), containing all sources and enclosed by a surface \( S \), and an open, source-free volume in which the point of observation \( P \), in which the field has to be determined, is located. It can be shown (e.g. Sommerfeld 1964, § 46) that the field in \( P \) can be found from an integration of the source-field values \( E_0 \) and \( H_0 \) over the volume \( V_1 \).

The magnetic field \( \mathbf{H} \) in point \( P \) follows from the definition of \( \mathbf{A} \) and Eq. (3.5)