1. INTRODUCTION

Support Vector Machines (SVM) are a good candidate for the solution of antenna array processing problems such as beamforming, estimation of angle of arrival or Ultra-Wide Band (UWB) electromagnetic design, because these algorithms provide superior performance in generalization ability and computational complexity. In this work we revise some applications of the SVM for antenna array processing and for target detection in UWB sea surface surveillance radar return profiles.

The first presented approach is based on the use of linear and nonlinear regressor schemes applied to antenna array processing, in particular to beamforming. The last one is related to the application of nonlinear multiclass Support Vector classification applied to radar object detection. Comparisons with conventional strategies and simulation results are provided to demonstrate the advantages of the Support Vector Machine approaches.

Since the 1990s there has been a significant activity in the theoretical development and applications of SVM. The first applications of machine learning have been related to data mining, text categorization, and other classical pattern recognition tasks. Recently, however, SVM have been applied to signal processing, wireless communication problems, notably spread spectrum receiver design and channel equalization.

The high-speed “learning” capability of SVMs is our main motivation for employing them in antenna problems. There are two approaches for the SVM optimization. The linear and the non-linear kernels are chosen depending on the nature of the problem. Unlike previous SVM algorithms in signal processing, in which real-valued
SVM algorithms were used to deal with complex valued data, we directly use a complex notation to solve the SVM optimization problem, which leads to functionals that are natural and straightforward extensions of the standard SVM dual optimization problems. This complex-value formulation is more suitable for handling UWB signal cases than some of the other algorithms.

2. COMPLEX SVM REGRESSOR FOR ANTENNA ARRAY PROCESSING

2.1. Temporal Reference

The temporal reference beamformer is intended to minimize the error between the output and a reference or desired signal \( d[n] \). Introducing the estimation error \( e[n] \), the expression for the output of the array processor can then be written as:

\[
y[n] = w^T x[n] = d[n] + e[n]
\]

where \( w = [w_1, ..., w_M]^T \) are the weights in the adaptive array.

For a set of \( N \) observed samples of \( \{x[n]\} \) and when nonzero empirical errors are expected, the functional to be minimized is:

\[
L_p = \frac{1}{2} \|w\|^2 + C \sum_{n=1}^{N} \ell(\xi_n, \xi_n) + C \sum_{n=1}^{N} \ell(\zeta_n, \zeta_n)
\]

with the constraints

\[
\begin{align*}
\text{Re}(d[n] - w^T x[n]) &< e + \xi_n \\
\text{Re}(-d[n] + w^T x[n]) &< e + \xi_n^* \\
\text{Im}(d[n] - w^T x[n]) &< e + \zeta_n \\
\text{Im}(-d[n] + w^T x[n]) &< e + \zeta_n^*
\end{align*}
\]

where \( \ell \) is a cost function. Equation (2) is the so-called SVR primal\(^4\,5\). The second half of Eq. (2) is intended to optimize the empirical error over the available sample set, where the first part is intended to minimize the norm of the weight vector. Provided that (as we will see later for the SVM in particular) the weight vector \( w \) is a linear combination of the input data\(^c\), this provides the solution which uses the minimum amount of energy of the input data.

\(^c\) Any linear algorithm that can be expressed as in Eq. (1) has a solution that can be expressed as a function of the data available for training. This has a straightforward proof if one assumes that the weight vector \( w \) lies in the subspace spanned by the available data.