18 Analysis after the 1740s

In the previous chapters of this second part, I described the growth of the theory of series from the 1720s to the 1750s. However, this evolution was part of a more general change in analysis which, during the 18th century, became an autonomous discipline, independent of geometry and arithmetic. This change matured in the 1730s and 1740s and was made manifest by the publication of Euler’s *Introductio in analysin infinitorum* in 1748.

In this chapter, I shall discuss the basic principles of the 18th-century concept of analysis, which lasted through to the first decades of the 19th century. The success and decline of the formal theory of series would be unintelligible if it was not considered within this context.

18.1 Eighteenth-century analysis as nonfigural and symbolic investigation of the real

In the preface to the *Institutiones calculi differentialis*, Euler made two remarkable observations about the nature of the differential calculus. First, he explicitly rejected geometrical confirmation as a means of testing the validity of the calculus, namely, he refused to accept proofs of the calculus’ correctness based solely on the fact that the calculus reached the same conclusions as elementary geometry: The calculus cannot have its own foundation in a geometrical reference [1755, 6]. He then observed:

> I mention nothing of the use of this calculus in the geometry of curved lines: that will be least felt, since this part has been investigated so comprehensively that even the first principles of the differential calculus are, so to speak, derived from geometry and, as soon as they had been sufficiently developed, were applied with extreme care to this science. Here, instead, everything is contained within the limits of pure analysis so that no figure is necessary to explain the rules of this calculus. (Euler [1755, 9; my emphasis])

Similar statements can be found in Lagrange’s writings. Indeed, in 1773, he wrote:

> I hope that the solutions I shall give will interest geometers both in terms of the methods and the results. These solutions are purely analytical and can be understood without figures. (Lagrange [1773, 661])

And, in his *Traité de mécanique analytique*, he stated:

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On 18th-century analysis, see Fraser [1989] and [1997].
One will find no figures in this work. The methods that I present require neither constructions nor geometrical or mechanical reasonings, but only algebraic operations, subject to a regular and uniform course. Those who admire analysis will with pleasure see mechanics become a new branch of it and will be grateful to me for having extended its domain (Lagrange [1788, 2]).

The insistence on figures can be easily understood if one thinks of the role that figures played in geometry (I refer to Chapter 7).\textsuperscript{258} In effect, when Euler and Lagrange claimed that figures were absent from their treatises, they were claiming the absence of inference derived by the mere inspection of a figure and therefore the independence of analysis from geometry, understood as a figural study of curves. This gives rise to a crucial question: What basic principles and instruments were used by 18th-century analysts to make analysis truly independent of geometry?

To answer this question, I shall begin by observing that d’Alembert considered the principles of analysis to be “based upon merely intellectual notions, upon ideas that we ourselves shaped by abstraction, by simplifying and generalising the ‘first’ ideas”.\textsuperscript{259} In other terms, analysis was considered as a system of merely intellectual notions, where the term “intellectual” referred to a form of knowledge that was not based on material awareness but was conceptual and mediated; it functioned in a discursive way along abstract notions. Whereas geometry was entrusted, to a certain extent, to the intuitive immediacy of an inspection of the figure and the perception of the relationships shown in the diagram, analysis was understood as a conceptual system where deduction was merely linguistic and mediated, or to put it another way, proceeded from one proposition to another discursively. Eighteenth-century analysis was not simply the linear continuation of Leibniz’s or Newton’s analysis but was based on a new way of doing mathematics. This new concept of analysis is undoubtedly closer to modern analysis than the previous one, even though it presents some aspects that significantly distinguish it from the modern concept. One of these aspects was the very notion of mathematical theory, which I shall examine in the remainder of this section.

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In Chapter 7, we already saw that the decisive aspect of analytical symbolism was the fact that signs were the concrete objects of a calculation,\textsuperscript{258} In Chapter 14, we also saw that this conception of the function of figures was still true for Euler.  
\textsuperscript{259}See d’Alembert [1773, 5:154]. By contrast, geometry and mechanics were “material and sensible” science; in particular, geometry was “the science of the properties of extension as it is considered as merely extended and figured” (d’Alembert [1773, 5:158]).