Models for routing a fleet of commercial vehicles

5.1 Introduction

In order to provide computer-based decision support, the real-life problem a motor carrier has to face must be represented by an analytical model, i.e. an abstraction of the real world which allows the automatic determination and evaluation of schedules. This chapter presents an overview of models for routing a fleet of commercial vehicles. The history of vehicle routing models dates back to 1959 when Dantzig and Ramser formulated the Truck Dispatching Problem. They described an application concerning the delivery of gasoline between a bulk terminal and several service stations supplied by the terminal. Since then, hundreds of models and thousands of algorithms have been proposed in the vehicle routing literature. These problems have in common the determination of the optimal set of tours to be performed by a fleet of vehicles \( V \) to serve a set of transportation requests \( O \). They generalise the well-known Travelling Salesman Problem (TSP) which is the problem of finding the least cost tour (for one vehicle) to visit a finite number of customers. The TSP is extensively described in Lawler et al. (1985) and has been introduced under the name Botenproblem (Messenger Problem) by Karl Menger in 1930 at a mathematical colloquium in Vienna. The TSP is known to be \( \mathcal{NP} \)-complete even if arc costs satisfy the triangle inequality. Thus, all of the problems discussed in this chapter are also \( \mathcal{NP} \)-complete as they generalise the TSP.

In road transport the movements of vehicles are restricted to the road network, the topology of which can be obtained from Geographical Information Systems. As the original road network is too complex to be considered directly, the original road network is usually transformed into a reduced network \((\mathcal{N},A)\). The nodes of the reduced network represent customer locations or the depot. In order to distinguish different customers at the same geographical location, the set of nodes may include

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1 See Dantzig and Ramser (1959)
2 See Menger (1932)
3 See Ahuja et al. (1993)
4 See section 2.2.3
different nodes referring to the same geographical location. For each pair of nodes \( n, m \in \mathcal{N} \) the arc \((n, m) \in \mathcal{A}\) is associated to the shortest path starting at node \( n \) and ending at node \( m \). Due to the presence of one-way streets and other regulations, the shortest path in the original road network usually depends on the direction of traversing a road and the type of vehicle. The set of arcs \( \mathcal{A} \subseteq \mathcal{N} \times \mathcal{N} \setminus \{(n, n) \mid n \in \mathcal{N}\} \) includes at most one arc from one node to another. Networks with multi-arcs\(^1\), i.e. several arcs in the same direction with different costs and travel times, can usually be considered with little additional effort.

In this work the term route is used for any path in the original road network representing the roads a vehicle is travelling on. The term tour is used to represent a sequence of nodes in the network \((\mathcal{N}, \mathcal{A})\) visited by a vehicle. Each tour is associated with a route which can be derived by replacing arcs between two successive nodes by the corresponding shortest paths in the original road network. Thus, finding the least cost route to cover a set of customer locations is equivalent to finding the least cost tour.

This chapter surveys several classical vehicle routing models and presents mathematical formulations of the models. It begins with a formulation of the Vehicle Routing Problem (VRP) which is the simplest and most studied model for routing a fleet of commercial vehicles. Then other classical models, namely the Vehicle Routing Problem with Time Windows (VRPTW), the Heterogeneous Fleet Vehicle Routing Problem with Time Windows (HFVRPTW), and the Pickup and Delivery Problem with Time Windows (PDPTW) are described.

Real-life vehicle routing problems encounter a variety of practical complexities which, to a certain extend, have been considered by the classical models. However, the classical models often oversimplify the problems that occur in practice. It appears, that in the vehicle routing literature more effort has been made in finding good solutions for the classical models, as for developing models that can handle the requirements occurring in real-life problems\(^2\). However, rich models, that can handle the requirements arising in real-life problems, are a fundamental prerequisite for sophisticated computer-based decision support. A model that can handle most of the real-life requirements, can significantly reduce the effort to manually verify an automatically generated schedule and the effort to transform the model recommendation into an applicable schedule. Hence, it can be concluded that the development of good models is equally important as the development of good algorithms for solving the problems.

This chapter introduces a general model that can handle the complexities evolving from various characteristics found in real-life vehicle routing problems that are not considered by the classical models. The model will be termed the General Vehicle Routing Problem (GVRP) and unifies the formulations of the classical vehicle routing problems as illustrated in figure 5.1.

Like the classical models, the GVRP assumes that travel times are constant, usually proportional to the distance travelled. In many real-life applications, however,

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\(^1\) See Ahuja et al. (1993)
\(^2\) See Kilby et al. (2000)