Chapter 19
Social Choice, Coase Theorem, Contracts, and Logrolling

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The problems connected with logrolling (Bernholz, 1974) and vote-trading (Kramer, 1973; McKelvey, 1976; Plott, 1967) are special cases of much wider phenomena (Bernholz, 1981; Schwartz, 1981, 1986). These phenomena are, in fact, the only reasons for the inconsistencies of nondictatorial societies described by Arrow’s General Impossibility Theorem (Arrow, 1963/51; see Sen, 1987, for a review of Social Choice Theory) “if more than one issues are implied, if individuals have separable preference orderings and if such inconsistencies are not present concerning the alternatives of single issues”. Moreover, the respective “paradoxes” of voting and logrolling, usually distinguished in the literature, are identical (Bernholz, 2000). Also, the implied social inconsistencies can occur even if participating individuals have identical preferences but face different restrictions (Breyer, 1980). Finally, the problems put into the center of attention by Arrow would not exist without the presence of negative externalities (in its broad sense, that is, including also political externalities) (Bernholz, 1982). But then, as suggested by the Coase Theorem, generalized below, stable Pareto-optimal outcomes do result in the absence of transaction costs in spite of the validity of Arrow’s Theorem, provided that binding contracts are possible. Consequently, the following statement by Sen (1987, p. 383) is true only if contracts are not binding: “... it would appear that there is no way of arriving at a social choice procedure specifying what is to be chosen..., satisfying the appropriately interpreted (i.e., in terms of choice) conditions specified by Arrow...”.

Definitions and Assumptions

To consider the general nature of the phenomena, consider a decentralized society, in which \( M = \{M_1, M_2, \ldots, M_n\} \) is the set of issues among whose alternatives humans can select. Each issue comprises at least two alternatives \( a_{ik(i)} \).

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(i = 1, 2, \ldots, n; k(i) = 1, 2, \ldots). An outcome is defined as containing one alternative out of each issue: \( a_g = (a_{1k(1)}, a_{2k(2)}, \ldots, a_{nk(n)}). \) Consequently, \( s = 1, 2, \ldots, q, \) where \( q = |M_1| \times |M_2| \times \ldots \times |M_n|. \) Further, let \( V_f \leq V \) denote the \( n \) subsets of society to which the rights to decide the \( n \) issues \( M_i \) have been assigned. \( V \) is the set of all adult people in society. We assume that these \( m \) individuals have weak, ordinal, complete, and transitive preferences over all outcomes. In some cases we will assume that individual preference orderings are separable. This means that if an individual prefers alternatives of one or a number of issues to other alternatives of the same issue(s), where the alternatives of all other issues remain constant, then this is also true for different alternatives of the other issues held constant. Formally, consider four different vectors \( a_{f}^h, a_{g}^{n-h}, a_{h}^n, a_{g}^n \) such that \( a_f \equiv (a_{f}^h, a_{f}^{n-h}) \), \( a_g \equiv (a_{g}^h, a_{g}^{n-h}) \). The four vectors contain \( h \) and \( n - h \) different alternatives, respectively, one out of each issue. Denote by \( R_j \) that individual \( j \) either prefers the first alternative to the second or is indifferent between them. Assume that \( a_{f}R_ja_{g} \). Then individual preferences are separable if also \( a_{g}R_ja_{f} \) holds. Or, similarly, if \( a_{f}R_ja_{g} \), then \( a_{g}R_ja_{f} \) is valid.

Logrolling and Cyclical Social Preferences

We prove first that a logrolling agreement beneficial to its participants always implies cyclical social preferences. It is assumed that individuals have complete, weak, transitive, and separable individual preference orderings (the separability assumption makes the proof easier, but it can be removed, see below). A logrolling situation is given if all decisions are taken by majority (or qualified majority) voting of the members of a group: if two or more subsets of this group who prefer