Waits and outcomes: What should be reported?

3.1 Introduction

This chapter presents statistical methods for the analysis of waiting-time data. In section 3.2 we introduce descriptive statistics that are used to summarize the frequency of events on wait lists. We provide the definition, estimation procedure, and interpretation for each summary measure, such as proportions, rates, cumulative incidence function. Measures of comparison and corresponding tests for comparing these measures statistically across groups are presented in section 3.3. Finally, section 3.4 describes regression models that are used to quantify the effects of explanatory variables on wait-list outcomes. For each model we show how to interpret the regression coefficients based on concepts from the previous two sections.

3.2 Descriptive statistics

3.2.1 Notation

Consider a wait-list population exposed to two competing risks, for example surgery, and death before surgery. Let event 1 be the primary event of interest and event 2 the competing event, and let \( T_m \) denote the latent time to event \( m = 1, 2 \). In this competing-risks setting, we observe the time to removal from the list \( T = \min\{T_1, T_2\} \) and the cause for removal \( \delta = \text{argmin}_m\{T_m\} \). If neither event occurs during the study period or the patient is lost to follow-up, the observation is considered censored at the time of removal from the wait list or the end of follow-up. Censored observations are included in the vector of observed times. The joint distribution of the random variables \((T, \delta)\) is of interest, where it is not assumed that the competing risks act independently.

Suppose the events occur at \( E \) distinct, unevenly spaced, ordered times \( t_1, t_2, \ldots, t_E \), and define \( t_0 = 0 \). Without loss of generality, we assume that time is measured in calendar weeks. Using Gooley’s notation, let \( e_i \) be the number of primary events at time \( t_i \), \( r_i \) be the number of competing events at time \( t_i \), \( c_i \) be the number of censored events at time \( t_i \), and \( n_i = n_{i-1} - (e_i + r_i + c_i) \) be the number of patients still waiting beyond time \( t_i \), where \( n_0 \) is the initial number of patients at risk [45].

We use this notation throughout this section. Although we consider only two types of events, the primary event and the competing event, events other than the primary event may be combined into the risk of a single competing event [91].
3.2.2 Proportions and odds

Definition

Proportions summarize the fraction of patients who experience a certain event. For instance, the weekly proportion of patients who were admitted, the proportion of patients who underwent surgery within target access times (TAT), and the proportion of patients who died before surgery are important indicators used to characterize access to elective surgery [114, 115, 116].

Although the proportion is the common measure for the probability of an event, an alternative is the odds of the event, which is defined as the ratio of the expected number of times that the event will occur to the expected number of times it will not occur [4]. For instance, the odds of death on the wait list by 52 weeks may be of interest. Unlike proportions, which are bounded by 0 and 1, the odds have a lower limit of 0 but no upper limit.

Estimation

To estimate the event proportion among waiting patients, \( p \), the number of events observed is divided by the total number of patients at risk:

\[
\hat{p} = \frac{\text{total number of events observed}}{\text{total number of patients at risk}}. \tag{3.1}
\]

For example, the estimated proportion of patients who were admitted for surgery during week \( i \) from the time of their registration on a wait list is estimated as

\[
\hat{p}_i = \frac{e_i}{n_{i-1}}. \tag{3.2}
\]

In this example, the event is surgery during week \( i \), and the patients at risk are those who were still on the wait list at the beginning of that week.

Another example is the proportion of patients who underwent surgery within the TAT, which is estimated by

\[
\hat{p} = \frac{1}{n_0} \sum_{\{i|t_i \leq \text{TAT}\}} e_i. \tag{3.3}
\]

Here, the event of interest is surgery within the TAT from registration, and the patients at risk are all those accepted on a wait list.

Similar quantities can be defined for events competing with the occurrence of surgery. The estimated proportion of patients who experienced the competing event, say preoperative death, within 52 weeks of registration is

\[
\hat{p} = \frac{1}{n_0} \sum_{\{i|t_i \leq \text{52 weeks}\}} r_i. \tag{3.4}
\]

In this example, the event of interest is death on the wait list within 52 weeks from registration, and the patients at risk are all those accepted for surgery.