Detection of Gaussian Signals in WGN

10.1 Introduction

Up to this point we have considered the detection of known or partially known signals in Gaussian noise. This type of situation is common in digital communications or active radar/sonar applications. However there exists other situations where the signal to be detected has a physical origin and is random. For example, in industrial applications, the onset of a malfunction in heavy machinery is often preceded by vibrations, which typically appear random, but nevertheless have a characteristic spectral signature. Similarly, in naval surveillance applications, the propeller noise of surface ships or submarines is typically random, but its spectral characteristics can often be used to identify not only the ship type, but even its model. In covert communications applications, to ensure that transmitted signals cannot be intercepted and demodulated by potential eavesdroppers, users often use spread spectrum modulation and encryption techniques. This produces waveforms that appear random to any user other than the intended one. From this perspective, the goal of a surveillance system becomes to detect the presence of such signals, and perhaps to characterize some of their features that might provide clues about the identity of covert users.

This chapter tackles therefore the detection of Gaussian signals in the presence of WGN. We consider both CT and DT signals, but since optimum receiver structures rely on waveform estimators, which are significantly easier to implement in the DT domain, we ultimately focus our attention on the DT case. The CT and DT detection problems can be treated in a unified manner by expressing them in the Karhunen-Loève coefficient domain. In Section 10.2, we show that the optimum detector can be implemented as a noncausal estimator-correlator receiver. This detector has a form reminiscent of the correlator/integrator receiver for detecting a known signal in WGN, except that the known signal needs now to be replaced by the estimate of the Gaussian signal to be detected based on observations in the interval $[0, T]$ over which measurements are available. Unfortunately, this estimate is noncausal, i.e.,
it is a smoothed estimate, since to obtain the estimate of the signal to be

detected at a time $t$, we need to wait until observations have been collected

over the entire interval. Fortunately, a rather ingenious receiver was devel-

oped independently by Schweppe [1] and Stratonovich and Sosulin [2] in the

mid-1960s, that relies entirely on a causal estimate of the signal. This receiver

architecture is described in Section 10.3. It has a structure remarkably simi-

lar to the correlator/integrator detector used for known signals in WGN and

can therefore be called the “true” extension of this detector to the Gaussian

signal case. Unfortunately, the performance of detectors for Gaussian signals

in WGN is rather difficult to characterize, since the probability distribution

of the sufficient statistic used by these detectors cannot be evaluated easily.

However, for the class of stationary Gaussian processes, it turns out that the

detector performance can be characterized asymptotically. Like the case of

i.i.d. measurements considered in Section 3.2, the asymptotic performance

analysis relies on the theory of large deviations [3,4]. However, instead of em-

ploying Cramer’s theorem, which is valid only for i.i.d. observations, it relies

on a powerful extension of this result, called the Gärtner-Ellis theorem, which

is applicable to dependent observations. In this context, the analog of the

Kullback-Leibler divergence for characterizing the rate of decay of the prob-

ability of a miss and of false alarm of NP tests is the Itakura-Saito spectral
distortion measure [5] between power spectral densities (PSDs) of stationary

Gaussian processes.

10.2 Noncausal Receiver

In Chapter 8 we considered the detection of known signals in Gaussian noise.

We start this chapter by examining a binary detection problem of the form

$$H_0 : Y(t) = V(t)$$

$$H_1 : Y(t) = Z(t) + V(t)$$

(10.1)

for $0 \leq t \leq T$, where the signal $Z(t)$ to be detected is zero-mean and Gaussian.

The signal $Z(t)$ has covariance

$$E[Z(t)Z(s)] = K(t, s) ,$$

(10.2)

and the noise $V(t)$ is a zero-mean WGN uncorrelated with $Z(t)$ and with
covariance

$$E[V(t)V(s)] = \sigma^2 \delta(t – s) .$$

(10.3)

As in the last two chapters, we consider simultaneously the DT and CT cases,

and thus the impulse $\delta(t – s)$ appearing in (10.3) denotes either a Kronecker or

a Dirac delta function, depending on whether we consider the DT or CT case.

In the CT case we assume that the covariance kernel $K(t, s)$ is continuous

over $[0, T]^2$. However, since optimal receivers rely on least-squares estimation

filters, we will ultimately focus on the DT case, since DT Wiener and Kalman

filters are much easier to implement than their CT counterparts.