AN INTERACTIVE ALGORITHM FOR DECOMPOSING: THE PARAMETRIC SPACE IN FUZZY MULTI-OBJECTIVE DYNAMIC PROGRAMMING PROBLEMS

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Abstract: The aim of this chapter is to study the stability of multi-objective dynamic programming (MODP) problems with fuzzy parameters in the objective functions and in the constraints. These fuzzy parameters are characterized by fuzzy numbers. For such problems, the concept and notion of the stability set of the first kind in parametric nonlinear programming problems are redefined and analyzed qualitatively under the concept of $\alpha$-Pareto optimality. An interactive fuzzy decision-making algorithm for the determination of any subset of the parametric space that has the same corresponding $\alpha$-Pareto optimal solution is proposed. A numerical example is given to illustrate the method developed in the chapter.

Key words: Fuzzy sets, Monte Carlo simulation, grey-related analysis, data mining

1. INTRODUCTION

Most practical vector optimization problems contain measured or estimated values that are represented by the different coefficients of the objectives and constraints. Such values may not be accurate enough to the errors in measuring, or estimating these values can lead to a false solution or a solution far from the exact solution of the considered problem. So, if
after solving the problem an error is discovered or some factors are 
changed that affect these coefficients, the problem has to be solved again.

Stability analysis covers this difficulty. It tells us what coefficients 
affect the solution greatly if they are changed and what coefficients have 
negligible effects on the solution.

In this chapter, we study the stability of multiobjective dynamic 
programming (MODP) problems with fuzzy parameters in the objective 
functions and in the constraints. These fuzzy parameters are characterized 
by fuzzy numbers. For such problems, concept and notion of the stability 
set of the first kind in parametric nonlinear programming problems are 
redefined and analyzed qualitatively under the concept of $\alpha$-Pareto 
optimality. An interactive fuzzy decision making algorithm for the 
determination of any subset of the parametric space which has the same 
corresponding $\alpha$-Pareto optimal solution is proposed. A numerical 
example is given to illustrate the method presented.

2. PROBLEM FORMULATION

In this chapter, the fuzzy multiobjective dynamic programming (FMODP) 
problem is considered. Fuzzy vector-minimization problem (FVMP) 
involving fuzzy parameters in the objective functions and in the constraints 
(see Abo-Sinna, 1998, 1992, 2004; Bellman, 1957; Bellman and Dreyfus, 
1962; Carraway et al., 1990; Chankong, 1981; Cohon, 1978; Deng Feng 
and Chuntian, 2004; Esogbue, 1983; Henig, 1983; Hussein and Abo Sinna, 
1993; 1995; Larson and Casti, 1978; 1982; Mangasarian, 1969; Osman and 
El-Banna, 1993; Saad, 1995; Su and Hsu, 1991; Tauxe et al., 1979) are 
selected:

FVMP:

\[
\begin{align*}
\text{Minimize} & \quad \{f_{q_1}(x_1, \tilde{a}_1), \ldots, f_{q_N}(x_N, \tilde{a}_N)\}, \quad q = 1, \ldots, Q, \quad Q \geq 2 \\
\text{subject to} & \quad G_m \left(g_{ml}(x_1, \tilde{b}_1), \ldots, g_{mN}(x_N, \tilde{b}_N)\right) \leq 0, \quad m = 1, \ldots, M, \\
& \quad x_n \in X_n, \quad n = 1, \ldots, N,
\end{align*}
\]