This chapter covers several more advanced topics in single machine scheduling. Some of these topics are important because of the theoretical insights they provide, others are important because of their applications in practice.

The first section considers a generalization of the total tardiness problem. In addition to tardiness costs, there are now also earliness costs; the objective functions are nonregular. The second section focuses on problems with a primary objective and a secondary objective. The goal is to first determine the set of all schedules that are optimal with respect to the primary objective; within this set of schedules a schedule has to be found then that is optimal with respect to the secondary objective. The third section also focuses on problems with two objectives. However, now the two objectives have to be considered simultaneously with the weights of the objectives being arbitrary. The overall objective is to minimize the weighted sum of the two objectives. The next section considers the makespan when there are sequence dependent setup times. There are two reasons for not having considered the makespan before. First, in most single machine environments the makespan does not depend on the sequence and is therefore not that important. Second, when there are sequence dependent setup times, the algorithms for minimizing the makespan tend to be complicated. The fifth section also considers sequence dependent setup times. However, now the jobs belong to a fixed number of different families. If in a
schedule a job is followed by a job from a different family, then a sequence
dependent setup time is incurred; if a job is followed by another job from the
same family, then no setup is incurred. A number of dynamic programming
approaches are described for various different objective functions. The sixth
section focuses on batch processing. The machine can process now a number of
jobs (a batch) simultaneously. The jobs processed in a batch may have different
processing times and the time to process the batch is determined by the longest
processing time. Various different objective functions are considered.

4.1 The Total Earliness and Tardiness

All objective functions considered in Chapter 3 are regular performance mea-
sures (i.e., nondecreasing in \(C_j\) for all \(j\)). In practice, it may occur that if job \(j\) is
completed before its due date \(d_j\) an earliness penalty is incurred. The earliness
of job \(j\) is defined as

\[
E_j = \max(d_j - C_j, 0).
\]

The objective function in this section is a generalization of the total tardiness
objective. It is the sum of the total earliness and the total tardiness, i.e.,

\[
\sum_{j=1}^{n} E_j + \sum_{j=1}^{n} T_j.
\]

Since this problem is harder than the total tardiness problem it makes sense
to first analyze special cases that are tractable. Consider the special case with
all jobs having the same due date, i.e., \(d_j = d\) for all \(j\).

An optimal schedule for this special case has a number of useful properties.
For example, it can be shown easily that after the first job is started, the \(n\) jobs
have to be processed without interruption, i.e., there should be no unforced
idleness in between the processing of any two consecutive jobs (see Exercise
4.1). However, it is possible that an optimal schedule does not start processing
the jobs immediately at time 0; it may wait for some time before it starts with
its first job.

A second property concerns the actual sequence of the jobs. Any sequence
can be partitioned into two disjoint sets of jobs and possibly one additional job.
One set contains the jobs that are completed early, i.e., \(C_j \leq d\), and the other
set contains the jobs that are started late. The first set of jobs is called \(J_1\) and
the second set of jobs \(J_2\). In addition to these two sets of jobs, there may be
one more job that is started early and completed late.

Lemma 4.1.1. In an optimal schedule the jobs in set \(J_1\) are scheduled first
according to LPT and the jobs in set \(J_2\) are scheduled last according to SPT.
In between these two sets of jobs there may be one job that is started early and
completed late.