The rotors studied in the previous chapters were assumed to be axially symmetrical. In the present chapter this assumption will be dropped, and more general models will be introduced to understand the effects of the lack of axial symmetry of the stator, the rotor, or both.

The assumption of axial symmetry of the whole machine is usually not verified exactly in real life, and sometimes an axi-symmetrical model can be only a very rough approximation of the actual system.

If a rotating machine is not isotropic about the rotation axis, the deviation from symmetry can concern only the rotor, only the stator, or both. In the first two cases, the model is not exceedingly complicated, but in the third case no exact solution of the relevant equations of motion can be found.

In some cases the system displays a nonisotropic behavior, even if all the parts of the machine are geometrically axi-symmetrical. This occurs particularly when the rotor runs on lubricated journal bearings: Under the effect of external forces the journal takes an off-center position within the bearing and reacts in different ways to the forces acting in the various planes including the rotation axis.

In the following sections the study of unsymmetrical rotors will be dealt with in subsequent steps. At first a very simple configuration, based on the Jeffcott rotor, will be studied. After the relevant phenomena have been qualitatively understood using this simplified model, a more complete
study, allowing quantitative results to be obtained even for complex systems, will be expounded.

25.1 Jeffcott rotor on nonisotropic supports

Consider the case of the rotor in Fig. 23.3b but assume that the stiffness of the supports is not isotropic in the $xy$-plane. All other assumptions made in Section 23.5.1, in particular the linearity of the system and the assimilation of the rotor to a point mass, will be retained. The motion will be studied in the $xy$-plane. In this plane the polar diagram of the stiffness of the supports is an ellipse, the so-called ellipse of elasticity.

Without any loss of generality, axes $x$ and $y$ will be assumed to coincide with the axes of the ellipse of elasticity, i.e., to be the principal axes of elasticity of the supporting structure and the stiffness along the $x$-direction to be lower than that along the $y$-direction. The elastic reaction of the shaft in this case is

$$F_x = -k_x x, \quad F_y = -k_y y.$$  \hfill (25.1)

By introducing the two different values of the stiffness into the equation of motion (23.9), the latter transforms into

$$\begin{aligned}
 m\ddot{x} + k_x x &= m\epsilon [\dot{\theta}^2 \cos(\theta) + \ddot{\theta} \sin(\theta)], \\
 m\ddot{y} + k_y y &= m\epsilon [\dot{\theta}^2 \sin(\theta) - \ddot{\theta} \cos(\theta)].
\end{aligned}$$  \hfill (25.2)

In the case of constant spin speed $\dot{\theta} = \Omega$, Eq. (25.2) with $\ddot{\theta} = 0$ still holds. From the homogeneous equation, it is clear that there are two natural frequencies, one (the lower) related to the motion in the $xz$-plane and the other related to the motion in the $yz$-plane:

$$\omega_{n_1} = \sqrt{\frac{k_x}{m}}, \quad \omega_{n_2} = \sqrt{\frac{k_y}{m}}.$$  \hfill (25.3)

They are not influenced by the spin speed, and then the Campbell diagram is made by two straight lines.

Remark 25.1 The motions in the two planes occur at different frequencies, so the two harmonic motions cannot combine to make circles or ellipses.

Remark 25.2 The fact that the two natural frequencies are independent of the spin speed causes the two critical speeds to coincide with the natural frequencies: $\Omega_{cr_1} = \omega_{n_1}$ and $\Omega_{cr_2} = \omega_{n_2}$.

At the first critical speed the motion reduces to a straight vibration along the $x$-axis, and at the other critical speed it reduces to a straight motion along the $y$-axis.