Forced Response in the Frequency Domain: Damped Systems

When damping is considered, the response of a linear system to a harmonic excitation is still harmonic in time, but is not in phase with the excitation. If damping is small, there is still a well-defined resonance (or many of them, depending on number of degrees of freedom), but its amplitude remains finite. If damping is large, one or more resonance peaks may disappear altogether.

7.1 System with a single degree of freedom: steady-state response

The response of a damped system with a single degree of freedom can be computed following the same lines seen in Chapter 6 for the undamped system. The excitation and the response can be written in the form

- Force excitation

\[ F(t) = f_0 e^{i\omega t}. \]  

(7.1)

- Excitation due to motion of the constraint

\[ x_A(t) = x_{A_0} e^{i\omega t}. \]  

(7.2)

- Response

\[ x(t) = x_0 e^{i\omega t}. \]  

(7.3)
As it was stated for the case of undamped systems, force $F(t)$ is a real quantity and should be expressed as $F = \Re(f_0 e^{i\omega t})$; in the same way, the expression of the displacements should mention explicitly the real part, since the complex notation is used to express quantities that have a harmonic time history as projections on the real axis of vectors that rotate in the complex plane. The symbol $\Re$ is, however, usually omitted.

Phasing is much more important for damped systems than for conservative ones, since damping causes the response to be out of phase with respect to the excitation. The amplitudes $f_0$ and $x_0$ are then complex quantities, with different phasing as shown in Fig. 7.1.

**Remark 7.1** The response to a harmonic excitation is harmonic, with the same frequency of the forcing function but out of phase with respect to the latter.

By introducing a harmonic time history for both excitation and response, the differential equation of motion can be transformed into an algebraic equation yielding the complex amplitude of the response

$$
\left( -m\omega^2 + i\omega c + k \right) x_0 = \begin{cases}
  f_0 , & (i\omega c + k)x_{A_0} , \\
  -m\omega^2 x_{A_0} ,
\end{cases}
$$

(7.4)

for excitation provided by a force, by the motion of the supporting point $A$ using an inertial coordinate, and by the motion of the supporting point $A$.

![FIGURE 7.1. Response of a system with viscous damping as seen in the complex plane.](image)