Chapter 3
Generalizing What We Learnt: Nonextensive Statistical Mechanics

Don Quijote me ha revelado íntimos secretos suyos que no reveló a Cervantes
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(Prólogo de Niebla de Miguel de Unamuno, 1935)

3.1 Playing with Differential Equations – A Metaphor

As we already emphasized, there is no logical-deductive procedure for generalizing any physical theory. This occurs through all types of paths that, in one way or another, are ultimately but metaphors. Let us present here a possible metaphor for generalizing the BG entropy.

The simplest ordinary differential equation can be considered to be

\[
\frac{dy}{dx} = 0 \quad (y(0) = 1).
\]

Its solution is

\[
y = 1,
\]

whose symmetric curve with regard to the bissector axis is

\[
x = 1.
\]

As the second simplest differential equation we might consider

\[
\frac{dy}{dx} = 1 \quad (y(0) = 1).
\]

Its solution is

\[
y = 1 + x,
\]
whose inverse function is
\[ y = x - 1. \tag{3.6} \]

We may next wish to consider the following one:
\[ \frac{dy}{dx} = y \quad (y(0) = 1), \tag{3.7} \]
whose solution is
\[ y = e^x. \tag{3.8} \]
Its inverse function is
\[ y = \ln x, \tag{3.9} \]
and satisfies of course
\[ \ln(x_A x_B) = \ln x_A + \ln x_B. \tag{3.10} \]

Is it possible to unify the three differential equations we considered up to now (i.e., (3.1), (3.4), and (3.7))? Yes indeed. It is enough to consider
\[ \frac{dy}{dx} = a + by \quad (y(0) = 1), \tag{3.11} \]
and play with the two parameters \(a\) and \(b\). Is it possible to unify the same three differential equations with only one parameter? Yes indeed, \ldots out of linearity! Just consider
\[ \frac{dy}{dx} = y^q \quad (y(0) = 1; q \in \mathbb{R}). \tag{3.12} \]
Its solution is
\[ y = [1 + (1 - q)x]^{1/(1-q)} \equiv e_q^x \quad (e_1^x = e^x). \tag{3.13} \]
Its inverse is
\[ y = \frac{x^{1-q} - 1}{1 - q} \equiv \ln_q x \quad (x > 0; \ln_1 x = \ln x), \tag{3.14} \]
and satisfies the following property:
\[ \ln_q(x_A x_B) = \ln_q x_A + \ln_q x_B + (1 - q)(\ln_q x_A)(\ln_q x_B). \tag{3.15} \]