3.1 The mean field model

This chapter outlines a somewhat different approach to the problem of tunable phase transitions in cognitive and other systems characterized by information sources. Here we use a ‘mean field’ approximation in contrast to the preceding chapter which can be considered a ‘mean number’ treatment. Hybrid models also seem possible.

Wallace and Wallace (1998, 1999) addressed how a language, in the broadest sense, ‘spoken’ on a network structure, responds as properties of the network change. The language might be speech, pattern recognition, or cognition. The network might be social, chemical, or neural. The properties of interest were the magnitude of ‘strong’ or ‘weak’ ties which, respectively, either disjointly partitioned the network or linked it across such partitioning. These would be analogous to local and mean-field couplings in physical systems. Here the system of interest is a set of information sources dual to cognitive processes which becomes linked through an average coupling by crosstalk.

Fix the magnitude of strong ties – again, those which disjointly partition the underlying network into cognitive or other submodules – but vary the index of nondisjunctive weak ties, \( P \), between components, taking \( K = 1/P \).

Assume the piecewise, adiabatically stationary ergodic information source depends on three parameters, two explicit and one implicit. The explicit are \( K \) as above and, as a calculational device, an ‘external field strength’ analog \( J \), which gives a ‘direction’ to the system. We will, in the limit, set \( J = 0 \). Note that there are many other ways of doing this, since classical renormalization techniques are more philosophy than prescription.

The implicit parameter, \( r \), is an inherent generalized ‘length’ characteristic of the phenomenon, on which \( J \) and \( K \) are defined. That is, \( J \) and \( K \) are written as functions of averages of the parameter \( r \), which may be quite complex, having nothing at all to do with conventional ideas of space. For example, \( r \) may be defined by the degree of niche partitioning in ecosystems or separation in social structures, and similarly for biological networks of various kinds.
For a given generalized language of interest having a well defined (adiabatically, piecewise stationary) ergodic source uncertainty, \( H = H[K, J, X] \).

To summarize a long train of standard argument (Binney et al., 1986; Wilson, 1971), imposition of invariance of \( H \) under some renormalization transform in the implicit parameter \( r \) leads to expectation of both a critical point in \( K \), written \( K_C \), reflecting a phase transition to or from collective behavior across the entire array, and of power laws for system behavior near \( K_C \). Addition of other parameters to the system results in a ‘critical line’ or surface.

Let \( \kappa \equiv (K_C - K) / K_C \) and take \( \chi \) as the ‘correlation length’ defining the average domain in \( r \)-space for which the information source is primarily dominated by ‘strong’ ties. The first step is to average across \( r \)-space in terms of ‘clumps’ of length \( R = < r > \). Then \( H[J, K, X] \rightarrow H[J_R, K_R, X] \).

Taking Wilson’s (1971) analysis as a starting point – not the only way to proceed – the ‘renormalization relations’ used here are:

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\chi(K_R, J_R) = \frac{\chi(K, J)}{R},
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(3.1)

with \( f(1) = 1 \) and \( J_1 = J, K_1 = K \). The first equation significantly extends Wilson’s treatment. It states that ‘processing capacity,’ as indexed by the source uncertainty of the system, representing the ‘richness’ of the generalized language, grows monotonically as \( f(R) \), which must itself be a dimensionless function in \( R \), since both \( H[K_R, J_R] \) and \( H[K, J] \) are themselves dimensionless. Most simply, this requires replacing \( R \) by \( R/R_0 \), where \( R_0 \) is the ‘characteristic length’ for the system over which renormalization procedures are reasonable, then setting \( R_0 \equiv 1 \). Length is measured in units of \( R_0 \).

Wilson’s original analysis focused on free energy density. Under ‘clumping,’ densities must remain the same, so that if \( F[K_R, J_R] \) is the free energy of the clumped system, and \( F[K, J] \) is the free energy density before clumping, then Wilson’s equation (4) is \( F[K, J] = R^{-3} F[K_R, J_R] \), so that

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Remarkably, the renormalization equations are solvable for a broad class of functions \( f(R) \), or more precisely, \( f(R/R_0), R_0 \equiv 1 \).

The second equation just states that the correlation length simply scales as \( R \).