Asset Price Bubbles in Complete Markets

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Summary. This paper reviews and extends the mathematical finance literature on bubbles in complete markets. We provide a new characterization theorem for bubbles under the standard no-arbitrage framework, showing that bubbles can be of three types. Type 1 bubbles are uniformly integrable martingales, and these can exist with an infinite lifetime. Type 2 bubbles are nonuniformly integrable martingales, and these can exist for a finite, but unbounded, lifetime. Last, Type 3 bubbles are strict local martingales, and these can exist for a finite lifetime only. When one adds a no-dominance assumption (from Merton [24]), only Type 1 bubbles remain. In addition, under Merton’s no-dominance hypothesis, put–call parity holds and there are no bubbles in standard call and put options. Our analysis implies that if one believes asset price bubbles exist and are an important economic phenomena, then asset markets must be incomplete.

Key words: Bubbles; no free lunch with vanishing risk (NFLVR); complete markets; local martingale; put–call parity; derivative pricing.

1 Introduction

Although asset price bubbles, their existence and characterization, have enthralled the imagination of economists for many years, only recently has this topic been studied using the tools of mathematical finance; see in particular Loewenstein and Willard [22], Cox and Hobson [7], Jarrow and Madan [20], Gilles [15], Gilles and Leroy [16], and Huang and Werner [17]. The purpose of this paper is to review and to extend this mathematical finance literature in order to increase our understanding of asset price bubbles. In this paper,
we restrict our attention to arbitrage-free economies that satisfy both the no-free-lunch-with-vanishing-risk (NFLVR) and complete markets hypotheses, in order that both the first and second fundamental theorems of asset pricing apply. Equivalently, there exists a unique equivalent local martingale measure. We exclude the study of incomplete markets. (We study incomplete market asset price bubbles in a companion paper, see Jarrow et al. [21].) We also exclude the study of charges, because charges require a stronger notion of no-arbitrage (see Jarrow and Madan [20], Gilles [15], and Gilles and Leroy [16]).

We make two contributions to the bubbles literature. First, we provide a new characterization theorem for asset price bubbles. Second, we study the effect of additionally imposing Merton’s [24] no-dominance assumption on the existence of bubbles in an economy. Our new results in this regard are:

(i) Bubbles can be of three types: an asset price process that is (1) a uniformly integrable martingale, (2) a martingale that is not a uniformly integrable martingale, or (3) a strict local martingale that is not a martingale. Bubbles of Type 1 can be viewed as the asset price process containing a component analogous to fiat money (see Example 2). Type 2 bubbles are generated by the fact that all trading strategies must terminate in finite time, and Type 3 bubbles are caused by the standard admissibility condition used to exclude doubling strategies.

(ii) Bubbles cannot be started—“born”—in a complete market. (In contrast, they can be born in incomplete markets.) They either exist at the start or not, and if they do exist, they may disappear as the economy evolves.

(iii) Bubbles in standard European call and put options can only be of Type 3, because standard options have finite maturities. Under NFLVR, any assets and contingent claims can have bubbles and put–call parity does not hold in general.

(iv) Under NFLVR and no-dominance, in complete markets, there can be no Type 2 or Type 3 asset price bubbles. Consequently, standard options have no bubbles and put–call parity holds.

The economic conclusions from this paper are threefold. First, bubbles of Type 1 are uninteresting from an economic perspective because they represent a permanent but stochastic wedge between an asset’s fundamental value and its market price, generated by a perceived residual value at time infinity.

Second, Type 2 bubbles are the result of trading strategies being of finite time duration, although possibly unbounded. To try to profit from a bubble of Type 2 or Type 3, one would short the asset in anticipation of the bubble bursting. Because a Type 2 bubble can exist, with positive probability, beyond any trading strategy, these bubbles can persist as they do not violate the NFLVR assumption. Type 3 bubbles occur in assets with finite maturities. For these asset price bubbles, unprotected shorting is not feasible, because due to the admissibility condition, if the short’s value gets low enough, the trading strategy must be terminated with positive probability before the bubble bursts. This admissibility condition removes downward selling pressure on the asset’s price, and hence enables these bubbles to exist.