Abstract

An approach for constructing a robust feedback control for problems having linear dynamics with disturbances is suggested. A useful control is assumed to be scalar and bounded.

The approach can be applied to conflict-controlled systems, where the constraint for the disturbance is unknown in advance. Adjustment of the method is based on results of the theory of linear differential games with fixed terminal time and geometric constraints for the players’ controls.

The algorithm for constructing the robust control is fulfilled as a computer program for the case when the quality of the process is defined only by two components of the phase vector at the terminal instant. The paper presents simulation results for the problem of lateral motion control of an aircraft during landing under wind disturbance.

Key words. Differential game, stable sets, robust control, switching surfaces, numerical constructions, aircraft landing, wind disturbance.
Introduction

In the theory of antagonistic differential games (see, for example, [18, 22, 3]), formulations are typical, where constraints both for the useful control and disturbance are given a priori. But in many practical situations, introducing the exact constraint for a disturbance is unnatural. For example, when a problem of aircraft landing is formulated, it is difficult to explain why the constraint on the possible deviation of the wind velocity from some average value is taken to be 10 m/sec, rather than, for example, 12 m/sec. With that, the optimal strategy obtained from the solution of a corresponding differential game depends on the taken disturbance level.

Let us agree that a control is called robust if, in the case of a “low” disturbance (which is unknown in advance), it provides good quality of the process by some “low” level useful control. With an increase in the disturbance level, the level of useful control guaranteeing good quality of the process grows too. This sense of the concept “robust control” coheres with that used in the mathematical literature (for example, see [14, 13, 38]).

An approach to constructing a linear robust control (for an $H^\infty$-problem) on the basis of the theory of differential games with linear-quadratic cost functional is described in [2].

This chapter suggests an alternative approach to constructing a nonlinear robust control. This method is oriented to problems with linear dynamics, where the constraint for the useful control is prescribed. The method is based on results of the theory of differential games with geometric constraints for the players’ controls.

The capacities of the method suggested are illustrated in the final part of the paper by a control problem of lateral motion of an aircraft during landing under wind disturbance. Its dynamics is described by a linearized system.

1 Construction of a Robust Control

1.1 Formulation of the Problem

Let us consider a linear differential game with fixed terminal time:

$$\dot{x} = A(t)x + B(t)u + C(t)v,$$

$$x \in \mathbb{R}^m, \quad t \in T, \quad u \in P = \{u \in \mathbb{R} : |u| \leq \mu\}, \quad v \in \mathbb{R}^q. \quad (1)$$

Here, $P$ is the constraint for the first player’s scalar control, and $T = [\theta_0, \theta]$ is the time interval of the game. The matrix functions $A$ and $C$ are continuous. The vector function $B$ is Lipschitzian in the time interval $T$.

The first player tries to lead system (1) to a set $M$ at the terminal instant $\theta$. The second player hinders this. The set $M$ is assumed to be a convex compactum in a subspace of $n$ chosen components of the vector $x$. Let us assume that the set $M$ includes a neighborhood of the origin of this subspace.