Aims and Objectives

- To introduce some theory of electromagnetic waves.
- To introduce optical bistability and show some related devices.
- To discuss possible future applications.
- To apply some of the theory of nonlinear dynamical systems to model a real physical system.

On completion of this chapter, the reader should be able to

- understand the basic theory of Maxwell’s equations;
- derive the equations to model a nonlinear simple fiber ring (SFR) resonator;
- investigate some of the dynamics displayed by these devices and plot chaotic attractors;
- use a linear stability analysis to predict regions of instability and bistability;
- plot bifurcation diagrams using the first and second iterative methods;
- compare the results from four different methods of analysis.
As an introduction to optics, electromagnetic waves are discussed via Maxwell’s equations.

The reader is briefly introduced to a range of bistable optical resonators, including the nonlinear Fabry–Perot interferometer, the cavity ring, the single fiber ring (SFR), the double-coupler fiber ring, the fiber double-ring, and a nonlinear optical loop mirror (NOLM) with feedback. All of these devices can display hysteresis and all can be affected by instabilities. Possible applications are discussed in the physical world.

Linear stability analysis is applied to the nonlinear SFR resonator. The analysis gives intervals where the system is bistable and unstable but does not give any information on the dynamics involved in these regions. To use optical resonators as bistable devices, the bistable region must be isolated from any instabilities. To supplement the linear stability analysis, iterative methods are used to plot bifurcation diagrams.

For a small range of parameter values, the resonator can be used as a bistable device. Investigations are carried out to see how the bistable region is affected by the linear phase shift due to propagation of the electric field through the fiber loop.

### 14.1 Maxwell’s Equations and Electromagnetic Waves

This section is intended to give the reader a simple general introduction to optics. Most undergraduate physics textbooks discuss Maxwell’s electromagnetic equations in some detail. The aim of this section is to list the equations and show that Maxwell’s equations can be expressed as wave equations. Maxwell was able to show conclusively that just four equations could be used to interpret and explain a great deal of electromagnetic phenomena.

The four equations, collectively referred to as Maxwell’s equations, did not originate entirely with him but with Ampère, Coulomb, Faraday, Gauss, and others. First, consider Faraday’s law of induction, which describes how electric fields are produced from changing magnetic fields. This equation can be written as

\[ \oint_C \mathbf{E} \cdot d\mathbf{r} = -\frac{\partial \phi}{\partial t}, \]

where \( \mathbf{E} \) is the electric field strength, \( \mathbf{r} \) is a spatial vector, and \( \phi \) is the magnetic flux. This equation may be written as

\[ \oint_C \mathbf{E} \cdot d\mathbf{r} = -\frac{\partial}{\partial t} \iint_S \mathbf{B} \cdot d\mathbf{S}, \]

where \( \mathbf{B} \) is a magnetic field vector. Applying Stokes’s theorem,

\[ \iint_S \nabla \wedge \mathbf{E} \cdot d\mathbf{S} = -\frac{\partial}{\partial t} \iint_S \mathbf{B} \cdot d\mathbf{S}. \]