12

Optimal Computer Control via Asynchronous Communication Channels

12.1 Introduction

In this chapter, we proceed with studying problems of optimal control via delayed, lossy, and asynchronous communication channels. In the previous chapter we addressed such a problem under the assumption that unlike the observation channels, the control loop is perfect, and so the controller output acts upon the plant immediately. Now we focus on the case where the control loop is delayed and lossy. As for the observations transmission and plant, the situation is just like in Chap. 11.

Specifically, we consider finite-horizon linear-quadratic optimal control problems for discrete-time partially observed systems perturbed by white noises. Data are sent from the sensors and controller to the controller and actuators, respectively, over parallel randomly delayed channels. Various signals are transferred with independent and a priori unknown transmission times. The signals may arrive out of order; there may be periods where no signal is received. The transmitted data may be lost due to, e.g., noise in the communication medium and protocol malfunctions.

We still suppose that any transmitted message is equipped with a time stamp indicating the moment of the transfer beginning. Hence the observations transmission times become known to the controller at the moments when the messages arrive at it. Likewise, the control signal transmission times become known at the actuators sites. We also suppose that this information is sent back, maybe, with delays and not continuously to the controller via special feedback control channels. As a result, there is an awareness about the bygone states of the communication medium, whereas its future states are unknown.

In this chapter, the solution of an analog of the classic finite-horizon linear-quadratic Gaussian (LQG) optimal control problem (see Appendix C starting on p. 509) is obtained for two different problem settings.

In the first one, the actuators are basically capable only to execute the currently received control signal. (In other words, they are not equipped with computing or memory modules.) So the (central) controller bears the entire responsibility for achieving the optimal performance. In this case, we suppose that this controller is given an additional information: The statistics of the data delays and dropouts in
the control channels is known in advance. However no such information is available for the channels carrying data from the sensors. Furthermore, we suppose that the feedback control channels do not drop data and provide delays not exceeding the sample period. Under certain technical assumptions, an optimal strategy to control the plant is obtained. It is shown that the optimal control results from feeding a linear feedback with a minimum variance state estimate, along with several past controls. To generate this estimate, the recursive state estimator from Subsect. 11.3.2 (starting on p. 375) is employed. Explicit formulas for the gain matrices of the optimal feedback are offered. The core of them is constituted by a finite set of coupled difference Riccati equations.

It should be remarked that many communication channels do not satisfy the time invariance condition, and a reliable prognosis of the future states of the communication medium is often a hard problem (see, e.g., [208], [232]). This is taken into account in the second setup of the LQG optimal control problem considered in this chapter. It is not assumed any longer that the statistics of data delays and dropouts is known in advance. Moreover, no restrictions on this statistics are imposed. Similarly, it is not assumed any longer that the delays in the feedback control channels do not exceed the sample period: These delays may be arbitrary. Another distinction with the previous problem setup is that now each actuator is endowed with a rather powerful computing module, which can be viewed as a local controller. The central controller remains in use. It collects data from the sensors and sends messages to the local controllers. We also suppose that the system disintegrates into multiple semi-independent subsystems. Each local controller serves its own subsystem, which do not interact. There also is an uncontrolled subsystem affecting all controlled ones.

We have in mind the situation where due to the limited bandwidths of the control channels, the local controllers cannot have access to the entire sensor data. This gives rise to the role of the central controller as a processor and compressor of these data. Ideally, this controller might generate the control for each subsystem. However its unawareness about the time that will be taken to transmit the control restricts its ability to achieve the best performance without an aid of the local controllers.

We show that this performance can be achieved via certain distribution of control functions between the central and local controllers. More precisely, we consider the performance best under the artificial assumption that all sensor data are resent from the central controller to the local ones. An outline of this distribution is as follows. Proceeding from the sensor data, the central controller forms a whole package of controls for any subsystem at each sampling time and sends these packages via the control channels. On arrival of such a package at a subsystem, its local controller chooses the proper member of the package, proceeding from its time stamp, and then corrects this member. The correction is generated recursively by the local controller. In doing so, it employs the information about the past actuator inputs for the corresponding subsystem but does not utilize the sensor data. As a result, the performance optimal in the idealized circumstances where the control channels bandwidths constraints are neglected is achieved without transmission the entire sensor data to the subsystems. The sizes of the control packages are determined explicitly. The crucial point is that under certain circumstances, these sizes may be far less than those in