The Phenomenology of Mathematical Beauty

Whereas painters and musicians are likely to be embarrassed by references to the beauty of their work, mathematicians enjoy discussions of the beauty of mathematics. Professional artists stress the technical rather than the aesthetic aspects of their work. Mathematicians, instead, are fond of passing judgment on the beauty of their favored pieces of mathematics. A cursory observation shows that the characteristics of mathematical beauty are at variance with those of artistic beauty. Courses in “art appreciation” are fairly common; it is unthinkable to find any “mathematical beauty appreciation” courses. We will try to uncover the sense of the term “beauty” as it is used by mathematicians.

What Kind of Mathematics Can Be Beautiful?

Theorems, proofs, entire mathematical theories, a short step in the proof of some theorem, and definitions are at various times thought to be beautiful or ugly by mathematicians. Most frequently, the word “beautiful” is applied to theorems. In the second place we find proofs; a proof that is deemed beautiful tends to be short. Beautiful theo-
ries are also thought of as short, self-contained chapters fitting within broader theories. There are complex theories which every mathematician agrees to be beautiful, but these examples are not the ones that come to mind in making a list of beautiful pieces of mathematics. Theories that mathematicians consider to be beautiful seldom agree with the mathematics thought to be beautiful by the educated public. For example, classical Euclidean geometry is often proposed by non-mathematicians as a paradigm of a beautiful mathematical theory, but I have not heard it classified as such by professional mathematicians.

It is not uncommon for a definition to seem beautiful, especially when it is new. However, mathematicians are reluctant to admit the beauty of a definition; it would be interesting to investigate the reasons for this reluctance. Even when not explicitly acknowledged as such, beautiful definitions give themselves away by the success they meet. A peculiarity of twentieth century mathematics is the appearance of theories where the definitions far exceed the theorems in beauty.

The most common instance of beauty in mathematics is a brilliant step in an otherwise undistinguished proof. Every budding mathematician quickly becomes familiar with this instance of mathematical beauty.

These instances of mathematical beauty are often independent of each other. A beautiful theorem may not be blessed with an equally beautiful proof; beautiful theorems with ugly proofs frequently occur. When a beautiful theorem is missing a beautiful proof, attempts are made by mathematicians to provide new proofs that will match the beauty of the theorem, with varying success. It is however impossible to find beautiful proofs of theorems that are not beautiful.

**Examples**

The theorem stating that in three dimensions there are only five regular solids (the Platonic solids) is generally considered to be beautiful; none of the proofs of this theorem, however, at least none of those that are known to me, can be said to be beautiful. Similarly, the prime number