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Contact Problems in Bending of Thermoelastic Plates

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12.1 Introduction

The theory of bending of plates with transverse shear deformation is very important in mechanical engineering because of its direct application to the study of deformable structures and because of its mathematical sophistication. The well-posedness of boundary value problems and of initial-boundary value problems with various types of boundary conditions for this model has been studied in detail in [ChCo00] and [ChCo05], respectively. Corresponding results for the same plate model where, additionally, there are significant thermal effects have been obtained in [ChEtAl04], [ChEtAl05a], [ChEtAl05b], [ChEtAl06], [ChCo07], [ChCo08a], [ChCo08b], [ChCo08c], [ChCo09a], and [ChCo09b]. Here we present the solution to the case of a piecewise homogeneous plate with transmission boundary conditions.

12.2 Formulation of the Problem

Suppose that the plate occupies a region $\bar{S} \times [-h_0/2, h_0/2]$, $\bar{S} \subset \mathbb{R}^2$. The displacement–temperature vector

$$
U(x, t) = (u(x, t)^T, u_4(x, t))^T, \quad x = (x_1, x_2) \in \bar{S},
$$

$$
u(x, t) = (u_1(x, t), u_2(x, t), u_3(x, t))^T
$$

satisfies the field equations

$$
LU(x, t) = B_0 \partial_t^2 U(x, t) + B_1 \partial_t U(x, t) + AU(x, t) = Q(x, t), \quad (x, t) \in S \times (0, \infty),
$$

where

$$
B_0 = \text{diag} \{ \rho h^2, \rho h^2, \rho, 0 \}, \quad h^2 = h_0^2/12,
$$

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\[ B_1 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \eta \partial_1 & \eta \partial_2 & 0 & -\nu^{-1} \end{pmatrix}, \quad A = \begin{pmatrix} A & h^2 \varpi \partial_1 \\ h^2 \varpi \partial_2 & 0 \\ \mu \partial_1 & 0 \\ 0 & -\Delta \end{pmatrix}, \]

\[ A = \begin{pmatrix} -h^2 \mu \Delta - h^2(\lambda + \mu) \partial_1^2 + \mu & -h^2(\lambda + \mu) \partial_1 \partial_2 & \mu \partial_1 \\ -h^2(\lambda + \mu) \partial_1 \partial_2 & -h^2 \mu \Delta - h^2(\lambda + \mu) \partial_2^2 + \mu & \mu \partial_2 \\ -\mu \partial_1 & -\mu \partial_2 & -\mu \Delta \end{pmatrix}, \]

\( \rho, \varpi, \eta, \kappa, \lambda, \) and \( \mu \) are physical constants, and \( \Delta \) is the Laplacian. Without loss of generality [ChEtAl04], we consider the initial conditions

\[ U(x, 0) = 0, \quad \partial_t u(x, 0) = 0, \quad x \in S. \]

We assume that the plate is infinite and piecewise homogeneous; that is, it is made of one material occupying an interior domain \( S^+ \) and of another one occupying an exterior domain \( S^- \). The two domains are separated by a simple, closed, smooth curve \( \partial S \) and such that \( \bar{S}^+ \cup \bar{S}^- = \mathbb{R}^2 \). Both materials are homogeneous and isotropic. We write

\[ \Sigma^\pm = S^\pm \times (0, \infty), \quad \Gamma = \partial S \times (0, \infty) \]

and consider the initial-boundary value problem (TC) consisting of

\[ L^\pm U^\pm(x, t) = Q^\pm(x, t), \quad (x, t) \in \Sigma^\pm, \]

\[ U^\pm(x, 0) = 0, \quad \partial_t u^\pm(x, 0) = 0, \quad x \in S^\pm, \]

\[ U^+_x(x, t) - U^-_x(x, t) = F(x, t), \quad (x, t) \in \Gamma, \]

\[ (T^+_U)_x^+(x, t) - (T^-_U)_x^-(x, t) = G(x, t), \quad (x, t) \in \Gamma, \]

where

\[ (T^\pm U^\pm)_x(x, t) = \begin{pmatrix} (T^\pm u^\pm)(x, t) - h^2_\pm \varpi n(x) u^\pm,4(x, t) \\ \partial_n u^\pm,4(x, t) \end{pmatrix}, \]

\[ T = \begin{pmatrix} h^2_\pm [(\lambda_\pm + 2\mu_\pm) n_1 \partial_1 + \mu_\pm n_2 \partial_2] & h^2_\pm (\lambda_\pm n_1 \partial_2 + \mu_\pm n_2 \partial_1) & 0 \\ h^2_\pm (\mu_\pm n_1 \partial_2 + \lambda_\pm n_2 \partial_1) & h^2_\pm [(\lambda_\pm + 2\mu_\pm) n_2 \partial_2 + \mu_\pm n_1 \partial_1] & 0 \\ \mu_\pm n_1 & \mu_\pm n_2 & \mu_\pm \partial_n \end{pmatrix}, \]

\( n = (n_1, n_2, 0)^T \) is the unit outward normal to \( \partial S \), \( \partial_n \) is the derivative in the direction of \( n \), and the subscripts and superscripts \( \pm \) distinguish between the constants, functions, and operators characterizing the domains \( S^+ \) and \( S^- \).

### 12.3 Function Spaces

We denote the Laplace transformation by