Continuum with Singularity

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Constitutive relations are mathematical relations between dual variables and primal variables. The essence of constitutive relations is that they define idealized materials. The histories of dual variables at any point \( M \) and any time \( t \) are determined by the histories of the primal variables in all points of the body up to the time \( t \). When only the \( n \)th order time derivatives of primal variables are involved instead of their entire histories, materials are said to be of the rate type \( n \), e.g., [194]. The goal of this chapter is to propose the basic framework for continuum with singularity distribution. The essential points of this chapter were developed in a previous paper [163].

4.1 Introduction

4.1.1 Principles for constitutive laws

As with classical continua, constitutive laws of continuum with distribution of singularity should account for behaviors such as elasticity, anelasticity, and viscosity. Physically, any internal process within materials has in principle a natural time. This time is defined as the measure of the time needed for the internal process to move to a new equilibrium after a change of the macroscopic external loading. The different mechanical behaviors of materials can then be classified as a function of their natural time scale, e.g., [158]:

L. R. Rakotomanana, *A Geometric Approach to Thermomechanics of Dissipating Continua*
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1. **Elasticity.** The medium is said to be elastic if the time scale of the observer, defined as the interval between two macroscopic typical observations, is infinitely smaller than any natural time of all internal processes in the medium. An elastic medium remembers entirely its previous state.

2. **Anelasticity.** The medium has a finite time memory (sometimes called anelastic) if the observer perceives the medium to return to the equilibrium state with a time delay. The time scale of the observer is of the same order of any natural time of all internal processes.

3. **Viscosity.** The medium has a long-term memory (sometimes called viscous) if it never returns to its original reference configuration meaning that the time scale of the observer is infinitely greater than any time scale of all internal processes. A viscous fluid does not remember its initial state.

In this book, we limit ourselves to elastic and viscous effects and then a priori neglect any influence of the finite time memory. However, introduction of two geometrical variables such as torsion and curvature may indirectly account for “some” history of deformation.

Following the idea of Coleman, a comprehensive theory of continuum with singularity may be based on the hypothesis that the dual variables, such as the stress tensor, heat flux vector, internal energy, and entropy, are determined by the histories of deformation and temperature. Constitutive functionals are allowed to depend on the \( \nabla \theta, \mathbf{N}, \) and \( \mathbf{R} \) as parameters (principle of determinism):

\[
\mathcal{Z} = \sum_{t=0}^{\infty} \left[ \omega_0(t - s), \mathbf{g}(t - s), \theta(t - s), \mathbf{N}, \mathbf{R}, \nabla \theta \right].
\]

They extend the usual thermo-viscoelasticity by including the histories of deformation and temperature [158]. They also replace the viscoelastic laws of the rate type by accounting for the effects of memory. The concept of a simple material was proposed by Noll in the fifties. The main principles used to formulate constitutive equations have been laid down by Noll in 1958, e.g., [146], [194], [201]:

1. **Principle of determinism:** Knowledge of the motion and temperature histories of all particles of the body is enough to determine the values of the dual variables of particle \( M \) at instant \( t \).

2. **Principle of material frame indifference:** A referential body with a clock defines an observer. If two observers consider the same motion and temperature histories in a given body, they find the same state for all dual variables. Basic roots underlying the principle of material frame indifference (objectivity) in relation with various invariance groups are recalled and discussed in Appendix B.