12 Fatigue and Incremental Collapse

12-0 INTRODUCTION

Cyclic fatigue is said to cause the failure of a structural member which breaks during a load cycle that it has previously withstood. This type of failure is perhaps the most common in machine elements subjected to cyclic stresses in the elastic range. In the pressure vessel industry, a similar type of elastic strain fatigue occurs when a component suffers rapid vibration, for instance in welded brackets supporting unbalanced rotating machinery, in pipes under pulsating flow, etc. The amplitude of the oscillations, compared to the steady value of the load, is usually small and fracture occurs after a large number of cycles, say $10^5$–$10^7$ cycles. Design to prevent this type of fatigue failure follows generally accepted rules and will not be discussed here in detail, but the reader is referred to standard books on the subject (Refs. 1, 2, 3).

The necessity to reduce the safety factors in order to achieve a more efficient use of the material, and the severity of the stresses set up during the rapid transients to which modern plant is subjected, have brought to light the problem of low cycle fatigue. In this case, failure occurs before about $10^4$–$10^5$ cycles, under applied stresses that are high enough to produce appreciable plastic deformations, i.e. corresponding to a stress range of about twice the yield stress. The mechanism of failure may be described as follows. Assume that the stress–strain curve in uniaxial tension for a given material is as shown in Fig. 12-1 and that cycling takes place between $+\varepsilon_{\text{max}}$ and $-\varepsilon_{\text{max}}$, following the hysteresis loop $OABCDA$. The total strain range is

$$\Delta\varepsilon_T = \Delta\varepsilon_e + \Delta\varepsilon_p \approx \Delta\varepsilon_p$$

where $\Delta\varepsilon_e$ is the elastic component and $\Delta\varepsilon_p$ the anelastic component. It is usually possible to neglect the elastic component, thus obtaining the diagram of Fig. 12-2. A rigid ideally plastic material, subjected to load cycles giving the same strain range as before and complete strain reversals, would follow $OA_1B_1C_1D_1A_1$. In this simple model, the difference between the strain energy absorbed by the ‘real’ material and the ideally plastic material is,
2 × \frac{1}{2} m (\Delta e_p)^2

per cycle. After \( N \) cycles, the energy becomes \( Nm(\Delta e_p)^2 \). It is assumed that

\[
\text{Stress}
\]

\[
\text{Strain}
\]

\[ -\varepsilon_{\text{max}} \quad \text{O} \quad +\varepsilon_{\text{max}} \]

\[ \Delta \varepsilon_{\varepsilon} + \Delta \varepsilon_p \]

**FIG. 12·1** Plastic cycling diagram.

\[ \text{Shaded area} \]

\[ \Delta \varepsilon_p \]

**FIG. 12·2** Simplified plastic cycling diagram.

the material can only absorb a certain amount of this energy (Ref. 4) and that failure occurs when

\[ Nm(\Delta e_p)^2 = k \]  

(12·1)