4

Electrical machine dynamics

4.1 INTRODUCTION

In this chapter, we shall look at the dynamic performance of some typical machines using approximate methods of solution. The analytical technique developed in the previous chapter leads to more exact solutions and will be used later, in the form of generalised machine theory. There is, however, the danger that by formulating equations in a 'mechanical manner' and solving them by using a digital computer, one may lose sight of the physical processes that are involved. The approximate methods often help us to obtain a physical insight into the behaviour of the systems.

The dynamical performance of electrical machines is determined by the interaction of all the mechanical forces acting on the rotating mass. Some of these forces may be impressed from sources outside the machine, some will be generated by the action of electrical current flowing in the various coils and the magnetic field in which the rotor is immersed. Clearly the polar moment of inertia of the total rotating structure will play an important part in the dynamical response of any machine. In steady state operation mechanical torque impressed on the rotating shaft will balance the torque of electromagnetic origin and in this equilibrium condition, in which the rotor is rotating at a constant velocity, the moment of inertia will play no part. As soon as the equilibrium condition is upset, however, for any reason, by a transient disturbance or superimposed small oscillation (hunting), there is a torque difference which will be balanced by forces arising from the kinetic energy stored in the rotor. This results in the appearance of the torque component $J\dot{\omega}$ acting upon the rotor, which will cause positive or negative acceleration, tending towards the restoration of equilibrium either at constant angular velocity or sustained 'hunting'.

In the following sections we shall consider the acceleration of a few types of machines when voltages are suddenly applied to them.
4.2 DYNAMICS OF D.C. MACHINES

4.2.1 Separately excited d.c. motor

Let us consider the starting of a separately excited d.c. motor. As has been explained earlier, d.c. motors are always started with reduced armature voltage and the applied voltage is increased as the back e.m.f. builds up with the speed. This particular motor may be started with or without load.

The voltage and torque equations of a separately excited d.c. motor are:

\[ V = R_a i_a + L \frac{d i_a}{dt} + K \Phi \omega \]  
\[ J \frac{d \omega}{dt} = K \Phi i_a - T_{\text{load}} \]

These are transient equations whereas those in Section 2.1.3 are for steady state.

It has been shown in equation (2.48) that the generated e.m.f. in a d.c. machine is

\[ E = N_a \Phi \omega \] (V)

In a motor, the generated e.m.f. is also called the back e.m.f. Here \( \Phi \) is the flux produced by the field winding. It penetrates the air gap and links \( N_a \) armature turns.

Following from Section 2.3, we can write

\[ \Phi = N_f i_f \mathcal{P}_f \]

We now have

\[ E = (N_a N_f \mathcal{P}_f) i_f \omega = M i_f \omega \]

and clearly \( (N_a N_f \mathcal{P}_f) \) is a rotational mutual inductance term as shown in Section 5.6. It must be pointed out here that this mutual inductance is not the same as that in coupled stationary circuits (Section 2.7). This term is responsible for generated voltage. We shall discuss this in greater detail in Chapter 5.

What we wish to emphasise here is that a stationary observer will measure a generated voltage \( E \) which is proportional to the field excitation \( i_f \) and the angular velocity \( \omega \) of the rotor. The constant of proportionality has the dimension of inductance. The observer will consider that the armature coil is fixed in space, because of the action of the commutator. Therefore, its inductance does not vary. In writing the transient equation for the armature we consider three terms.

\[ R_a i_a = \text{armature voltage drop} \]

\[ L_a \frac{d i_a}{dt} = \text{inductive voltage which disappears as soon as steady state current is reached} \]

\[ M i_f \omega = \text{back e.m.f. or generated e.m.f.} \]