UNIT A  INDICES

A Power is the result obtained by multiplying a number by itself a stated number of times.

e.g.  $2 \times 2 \times 2$ is called the third power of 2.

An Index (plural Indices) is a small figure written to the right of a number, and slightly above it, to indicate the power to which the number is raised.

e.g.  $2 \times 2 \times 2 = 2^3$, the 3 being the Index.

A root of a number is such an amount that, when multiplied by itself a stated number of times, it will produce the given number.

e.g.  49 is the second power, or square of 7  
and  7 is the square root of 49  
81 is the fourth power of 3, since $3^4 = 81$

So 3 is the fourth root of 81

A root is indicated by a radical ($\sqrt{}$) with a small number to the left.

Thus $\sqrt[4]{81} = \text{the 4th root of 81} = 3$.

In the case of square roots, the small 2 is omitted.

So $\sqrt{49}$ is the square root of 49, i.e. 7.

An Index Number may be:  (a) Positive, e.g. $2^7$;  (b) Negative, e.g. $2^{-3}$;  (c) Fractional, e.g. $2^{\frac{1}{2}}$ or $3^{-\frac{2}{5}}$. 
Laws of Indices

1 Multiplication.

\[ 2^2 \times 2^3 = (2 \times 2) \times (2 \times 2 \times 2) = 2^{2+3} = 2^5 \]

The rule is to add the indices of the same factor.

2 Division.

\[ 2^5 \div 2^3 = \frac{2 \times 2 \times 2 \times 2 \times 2}{2 \times 2 \times 2} = 2 \times 2 \text{ (by cancellation)} \text{ or } 2^{5-3} = 2^2 \]

So the rule is to subtract the index of the denominator from that of the numerator.

3 Further powers.

When a factor is raised to a Power, it may be raised to a higher power by multiplying the two indices.

\[ (2^2)^3 = 2^2 \times 2^2 \times 2^2 = 2^{2 \times 3} = 2^6 \]

Negative Indices

These obey the same laws as above.

Multiplication—add the Indices.

\[ 3^{-2} \times 3^{-5} = 3^{(-2-5)} = 3^{-7} \]
\[ 3^5 \times 3^{-2} = 3^{5-2} = 3^3 \]

Division—subtract the Indices.

\[ 7^{-5} \div 7^{-3} = 7^{-5+3} = 7^{-2} \]
\[ 7^3 \div 7^{-2} = 7^{3+2} = 7^5 \]

Note. The student must watch the signs (+ or −) when dealing with negative values. In subtracting the index number of the divisor, the sign must be changed, because

\[ -(−3) = +3 \text{ and } −(−2) \text{ is } +2 \]

Powers—multiply the Indices.

\[ (5^{-4})^2 = 5^{-4 \times 2} = 5^{-8} \]
\[ (3^{-2})^{-3} = 3^{-2 \times -3} = 3^6. \quad (−) \times (−) = +. \]

Meaning of Negative Indices

Suppose we divide \( a^2 \) by \( a^5 \). We can do it two ways:

1 \[ a^2 \div a^5 = a^{2-5} = a^{-3} \text{ (rule of Indices) } \]