PLASTIC ANALYSIS OF FLANGE WRINKLING IN AXISYMMETRICAL DEEP-DRAWING

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SUMMARY

Flange wrinkling in axisymmetrical deep-drawing occurs as a result of compressive circumferential stresses and it is an undesirable mode of deformation. Flange does not remain stable and plastically buckles into a number of waves, endangering the success of the process and the quality of the product. Wrinkling can be prevented by the use of blank-holders constraining the plastic flow of the material.

In the present work, experiments are conducted to study the mode of plastic deformation and to compare with theoretical results. An analysis is developed based on the plasticity theory using the energy approach. It is found that wrinkling takes place when the plastic work associated with radial-drawing is greater than that in wrinkling. Based on the number of waves, the blank-holder load can be computed to prevent wrinkling.

NOMENCLATURE

- A, B, n: Constants in the strain-hardening equation
- b: Width of element of wave in wrinkling
- D: Blank diameter
- d_d: Die throat diameter
- d_p: Punch diameter
- H: Blank-holder load
- K: R_o/R_c
- L.D.R.: Limiting Drawing Ratio = D_o/d_p
- L: Half wave length
- M: Plastic bending moment
- N: Number of waves
- R: Radius on undeformed blank
- r: Radius on deforming blank
- V: Volume of flange
- W: Plastic work
- w: Plastic work per unit volume
- x: Coordinate in circumferential direction
- y: Deflection of wave
- Y: Yield stress
- z: Thickness coordinate in bending of the wave
- s: Arc length of quarter wave
- d_l: Factor of proportionality in plastic stress-strain equations
- γ: Maximum wave amplitude
- ε_1, ε_2, ε_3: Principal plastic strains in radial, circumferential and thickness directions
- ε: Equivalent strain
- p: Radius of curvature
- φ: Change of slope of wrinkled wave
- ε_1, ε_2, ε_3: Principal stresses in radial, circumferential and thickness directions
- σ: Equivalent stress

Subscripts

- b: Bending
- c: Point of start of die curvature
- d: Deep-drawing
- o: Original state
- m: Designation of points on blank (See Fig. 3)
- i, j: Suffixes of stress and strain tensor with range 1-3
- t: Total
- w: Wrinkling

Superscript

- *: Following an element during its strain history

INTRODUCTION

In the deep-drawing of a round blank by a cylindrical punch, stress and strain distributions in regions over the punch and die are quite different. As shown in Fig. 1, over the punch head biaxial...
tensile stresses are developed which cause thinning of the material. Over the die, in the radial direction tensile stresses act whereas in the circumferential direction compressive stresses are generated during drawing. Between the die and punch there is a transition between the two regions. The limiting drawing ratio (L.D.R.) which can be obtained depends on the load carrying capacity of the region above the punch head which in turn is governed by the tensile plastic instability taking place in this region. The complete solutions including the tensile plastic instability analyses for the axisymmetric deep-drawing problem have been obtained earlier [1,2].

The compressive circumferential stresses which are generated in the flange can cause plastic buckling of the blank and as a result waves are produced as shown in Fig.2. This undesirable mode of deformation is known as wrinkling and it can be prevented by the use of blank-holders as shown in Fig.1. A lateral force can be applied by the blank-holder plate which would not allow the development of waves in the lateral direction. In cases when this facility is not available in the press, then resort can be made to a clearance type of blank-holder. In this case the development of the waves are prevented by the fixed clearance between the die and the blank-holder and the elasticity of the assembly also contributes to the prevention of wrinkling by applying a spring-like force. Other blank-holder designs are possible employing springs. Drawbeads are also used in deep-drawing which force the material to go through a groove between the die and blank-holder thereby increasing the restraining force which change the stress distribution towards a more tensile one, thereby reducing the wrinkling behaviour [3].

It is therefore of great significance to the press and die designers to know whether a given blank will wrinkle or not, and if it wrinkles whether it will remain within acceptable limits. The tools will be designed accordingly choosing the appropriate type of blank-holder and using a sufficient force to prevent wrinkling. If a higher blank-holder force is used than necessary, then the L.D.R. in deep-drawing will decrease, and thinning of the rim will take place due to compression near the end of the draw.

First theoretical analysis of the flange wrinkling problem without a blank-holder has been attempted by Geckeler [4]. He developed an expression giving the critical stress at which buckling will occur. When the circumferential stress in the flange reaches a critical value calculated as functions of the plastic buckling modulus, thickness and flange width, buckling would occur. His second equation, \(N=1.65(R_d+R_c)/(R_o-R_c)\) gives the number of waves produced without a blank-holder. Baldwin and Howald [5] carried out an experimental investigation of the wrinkling problem and compared their results with those of Geckeler. Their results indicate that wrinkling starts after about 10% reduction in blank diameter. Theoretical and experimental results found in the present research shows that wrinkling can start right at the beginning of deep-drawing. Senior [6] extended Geckeler’s analysis and developed a theory for the cases of wrinkling under both constant and spring-loaded blank-holders. In his solution, energy expended on the flange by the circumferential stresses is equated to the energy dissipated in buckling the flange. A pressure distribution is assumed to simulate the fixity of the inner edge which seems to be rather unrealistic. A small deflection analysis is carried out using the plastic buckling modulus, and the flange is approximated by a number of linked struts. This analysis does not allow the prediction of number of waves under blank-holder load. A critical appraisal of the subject is given by Alexander [7].

As seen, the previous work on the subject is based on small deflection analyses not using the plasticity theory, and the mechanism of wrinkling is not fully understood. The double-modulus is used which involves drawing tangents to the experimentally obtained stress-strain curves introducing a source of errors.

The present research is based on the plasticity theory, taking into account large deformations and non-linear material behaviour. It is attempted to explain the mechanism of wrinkling and provide solutions for the applications with and without blank-holders.

**THEORY**

In any deformation process in the plastic range, materials try to deform in such a mode as to minimize the associated plastic work. In deep-drawing, in the flange region there are two possible modes of deformation, (i) drawing or (ii) wrinkling. Considering also the effects of constraints such as the blank-holder load, clearance and drawbeads, the blank will tend to deform in such a mode which requires less energy compared to the second mode.

To be able to study the problems of deep-drawing and wrinkling from the point of view of energy dissipation or plastic work the following analysis is conducted:

**Basic Equations**

In the analysis of the deep-drawing and wrinkling problems the elastic strains are not considered since they are negligible compared to the plastic strains. The total deformation is therefore equivalent to the plastic deformation. Since this is the case, the deformation must obey the basic equations of plastic flow given below [8]:

**Yield Criterion:**

For an isotropic strain-hardening material Von Mises yield criterion is used. When expressed in terms of the principal stresses:

\[
\sigma_1 - \sigma_2 + \sigma_2 - \sigma_3 + \sigma_3 - \sigma_1 = 2\tau^2
\]  

(1)