3

Production with a Surplus – Subsistence Wage

3.1 Introduction

It is now time to relax at least one of the assumptions of Chapter 2 – that the gross outputs of the economy are just enough to ensure reproduction and that the wage is at subsistence level. Clearly, the latter cannot be modified first. For, if the wage were raised above subsistence level but the technical coefficients were initially such that reproduction was just possible, the economy would be unable to produce enough to meet requirements: that is, more of both commodities would be used up than produced. So, it is necessary to relax, first of all, the assumption that the economy produces levels of gross output just sufficient to meet inter-industry requirements. Retaining the assumption that the wage is fixed (at subsistence level), advanced and incorporated into the input–output coefficients, I have as the two production processes:

\[ \tilde{X}_{11} \text{ qr wheat } \oplus \tilde{X}_{21} \text{ t iron } \oplus \tilde{X}_{1} \text{ qr wheat} \quad (3.1a) \]
\[ \tilde{X}_{12} \text{ qr wheat } \oplus \tilde{X}_{22} \text{ t iron } \oplus \tilde{X}_{2} \text{ t iron} \quad (3.1b) \]

where now:

\[ \tilde{X}_{1} \geq \tilde{X}_{11} + \tilde{X}_{12} \quad (3.2a) \]
\[ \tilde{X}_{2} \geq \tilde{X}_{21} + \tilde{X}_{22} \quad (3.2b) \]

with at least one strict inequality.\(^1\) Contrast these expressions with (2.1) and (2.2) respectively of Chapter 2. In the previous chapter,
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gross output levels were necessarily fixed from period to period. Now, if the economy is capable of producing a surplus, the possibility of accumulation arises. As I shall discuss growth in Chapter 7, I assume now that any surplus product is consumed. The system to be analysed in this chapter resembles that in the previous chapter, in the sense that gross output levels are fixed from period to period: the circulating capital used in production is replaced but there is no net investment. To ensure the absence of all dynamic elements from the economy, I also assume that the labour force is constant through time.

I am interested in characterising an economy capable of producing a physical surplus (over and above inter-industry requirements), or positive net output, of both commodities. This does not mean that the economy has to produce a net output of both commodities, only that it can do so. To analyse this particular question, I define, first of all, the net output of commodity $i$ by:

$$
\tilde{F}_i = \tilde{X}_i - \tilde{X}_{i1} - \tilde{X}_{i2} \quad i = 1,2
$$

(3.3)

$\tilde{F}_i$ is also referred to as the final demand for commodity $i$. The input-output coefficients are defined analogously to (2.3) of Chapter 2:

$$
\tilde{a}_{ij} = \tilde{X}_{ij}/\tilde{X}_j \quad i,j = 1,2
$$

(3.4)

Using (3.3), the inequalities (3.2) can be rewritten as equations; after substituting from (3.4), I obtain:

$$
\tilde{X}_1 = \tilde{a}_{11}\tilde{X}_1 + \tilde{a}_{12}\tilde{X}_2 + \tilde{F}_1
$$

(3.5a)

$$
\tilde{X}_2 = \tilde{a}_{21}\tilde{X}_1 + \tilde{a}_{22}\tilde{X}_2 + \tilde{F}_2
$$

(3.5b)

Equation (3.5a) states that the gross output of wheat, $\tilde{X}_1$, is divided between replacement of means of production in both wheat and iron industries ($\tilde{a}_{11}\tilde{X}_1 + \tilde{a}_{12}\tilde{X}_2$) and consumption ($\tilde{F}_1$); similarly for the gross output of iron in (3.5b).

3.2 The Quantity System

I wish to find conditions under which (3.5) has solution $\tilde{X}_1 > 0, \tilde{X}_2 > 0$, given any $\tilde{F}_1 > 0, \tilde{F}_2 > 0$. An economy capable of producing