Sampled-Data Signals

4.1 Introduction

Sampled-data signals have defined values only at certain instants of time, and arise whenever continuous functions are measured or recorded intermittently. In recent years such signals have come to assume great importance because of developments in digital electronics and computing. Since it is not possible to feed continuous data into a digital (as opposed to analogue) computer, any signal or data input must be represented as a set of numerical values. In almost every case the numbers represent sampled values of the continuous signal at successive equally-spaced instants. An example of a sampled-data signal of this type has already been illustrated in figure 1.2, which shows successive values of midday temperature measured in a particular place. A further example, in which successive samples represent values of an electrical potential between two points in a circuit, is shown in figure 4.1.

![A sampled-data signal](image)

Figure 4.1  A sampled-data signal

It should be emphasised from the start that a set of sample values only forms an adequate substitute for the underlying continuous signal waveform if the interval between successive samples is sufficiently small. This matter is discussed in some detail in chapter 8 where sampling and resynthesis of a continuous function from its sample values are considered as signal processing operations. The object of this chapter is limited to showing how sampled-data signals may be expressed mathematically, and how the foregoing framework of frequency-analysis techniques applies to them.
4.2 Mathematical description using the Dirac (δ) function

The Dirac function, often referred to as the impulse function, is a pulse of extremely short duration and unit area. In other words the product of its duration and its mean height is unity, even though its precise shape is undefined. The physical significance of such a function may be illustrated by an example. Suppose a mechanical impulse is delivered to a golf ball by the head of a golf club. Other things being equal, the momentum imparted to the ball and the distance it travels depend upon the value of the impulse, which is given by the product of the force and the time for which it is exerted. Or, assuming that the force is not constant, it is the area under the force–time graph which determines the impulse. Since the Dirac function is defined by its unit area, it may be used to denote a unit mechanical impulse, and we shall use the same function later in this text to describe a sudden electrical disturbance applied to a signal-processing device.

The Dirac function is also useful for describing a sampled data signal, which may be considered to consist of a number of equally-spaced pulses of extremely short duration. It is convenient in this case to think of all the signal samples as pulses of identical duration, with their heights proportional to the values of the signal at the relevant instants. In practice, this approach proves mathematically sound, provided the pulse duration is negligible compared with the interval between successive samples.

In order to discuss the spectrum of a sampled-data signal, it is necessary to evaluate that of the unit Dirac function. Using the convention that the symbol δ(t) represents a Dirac pulse occurring at t = 0, we have

$$G(\omega) = \int_{-\infty}^{\infty} \delta(t) \cdot e^{-j\omega t} \cdot dt$$

To evaluate this integral we make use of the so-called ‘sifting property’ of the Dirac function which is illustrated in figure 4.2. Here a function f(t) is shown multiplied by a unit Dirac function occurring at the instant t = a which is denoted by the symbol \(\delta(t - a)\). Since the area of the Dirac pulse is unity, the area under the curve representing the product \([f(t) \cdot \delta(t - a)]\) is the value of f(t) at \(t = a\).

When the product is integrated between \(t = -\infty\) and \(\infty\), the result is just this area. Hence the sifting property may be formally stated as follows

$$\int_{-\infty}^{\infty} f(t) \cdot \delta(t - a) \cdot dt = f(a)$$

The spectrum \(G(\omega)\) of a Dirac pulse at \(t = 0\) is therefore simply equal to the value of \(e^{-j\omega t}\) at \(t = 0\), which is unity. Hence

$$G(\omega) = 1$$

This result tells us that all frequencies are equally represented by cosine components. When a large number of cosines of similar amplitude but different frequency are added together, they tend to cancel each other out everywhere except at \(t = 0\),