KINEMATIC MAPPING APPLICATION TO APPROXIMATE TYPE AND DIMENSION SYNTHESIS OF PLANAR MECHANISMS

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Abstract Kinematic mapping is used for preliminary development of an algorithm for the approximate synthesis of planar four-bar mechanisms for rigid body guidance. Both dyad type and dimensions are determined. Planar mechanism coupler motions are represented as the curves of intersection of a pair of quadric constraint surfaces, one for each of two dyads. The problem reduces to identifying the two best constraint surfaces in the pencil of quadrics containing the curve. The overdetermined synthesis equations are linear in the unknown surface shape coefficients, and their products. Non-trivial solutions exist only in the nullspace of the coefficient matrix. While the algorithm remains incomplete, results presented herein are encouraging.

Keywords: Approximate type and dimensional synthesis, rigid body guidance, kinematic mapping, singular value decomposition.

1. Introduction

The kinematic synthesis of robot mechanical systems has been the focus of a significant volume of research; in particular, robots whose mechanical systems are parallel kinematic chains. For example Shirkhodaie and Soni, 1987 use a reasonably straightforward Cartesian approach for kinematic synthesis of planar parallel robots with three degrees-of-freedom (DOF). Murray and Pierrot, 1998 present an algebraic algorithm, based on quaternions, for synthesis of planar three-legged platforms given n-positions. We intend to develop an n-pose synthesis algorithm for both dyad type and dimensions based on the geometry of the kinematic image of the desired coupler positions and orientations. But,
as with many things kinematic, while the proposed concept is elegantly simple, the devil is in the details.

2. Kinematic Mapping

Kinematic mapping was introduced independently by Blaschke, 1911, and Grünwald, 1911. One can consider the relative displacement of two rigid-bodies in the plane as the displacement of a Cartesian reference coordinate frame $E$ attached to one of the bodies with respect to a Cartesian reference coordinate frame $\Sigma$ attached to the other. Without loss of generality, $\Sigma$ may be considered fixed with $E$ free to move, as is the case with the four-bar mechanism illustrated by Figure 1.

![Figure 1. Planar RRRP linkage.](image)

The homogeneous coordinates of points represented in $E$ are given by the ratios $(x : y : z)$. Those of the same points represented in $\Sigma$ are given by the ratios $(X : Y : Z)$. The position of a point $(X : Y : Z)$ in $E$ in terms of the basis of $\Sigma$ can be expressed compactly as

$$
\begin{bmatrix}
  X \\
  Y \\
  Z
\end{bmatrix} =
\begin{bmatrix}
  \cos \varphi & -\sin \varphi & a \\
  \sin \varphi & \cos \varphi & b \\
  0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
  x \\
  y \\
  z
\end{bmatrix},
$$

where the pair $(a, b)$ are the $(X/Z, Y/Z)$ Cartesian coordinates of the origin of $E$ expressed in $\Sigma$, and $\varphi$ is the orientation of $E$ relative to $\Sigma$, respectively.

The essential idea of kinematic mapping is to map the three homogeneous coordinates of the pole of a planar displacement, in terms of $(a, b, \varphi)$, to the points of a three dimensional projective image space. The image space coordinates are defined as:

$$
X_1 = a \sin(\varphi/2) - b \cos(\varphi/2); \quad X_3 = 2 \sin(\varphi/2)
$$

$$
X_2 = a \cos(\varphi/2) + b \sin(\varphi/2); \quad X_4 = 2 \cos(\varphi/2).
$$

The mapping is injective, not bijective: there is at most one pre-image for each image point. Any image point on the real line $l$, defined by the intersection of the coordinate planes $X_3 = X_4 = 0$, has no pre-image and therefore does not correspond to a real displacement of $E$. See Bottema and Roth, 1979, for a detailed analysis of the geometry of the image space.