“Deborah numbers”, coupling multiple space and time scales and governing damage evolution to failure

Y.L. Bai\textsuperscript{a}, H.Y. Wang\textsuperscript{a,*}, M.F. Xia\textsuperscript{b,a}, F.J. Ke\textsuperscript{c,a}

\textsuperscript{a} LNM, Institute of Mechanics, Chinese Academy of Sciences, Beijing 100080, China
\textsuperscript{b} Department of Physics, Peking University, Beijing 100871, China
\textsuperscript{c} Department of Applied Physics, Beijing University of Aeronautics and Astronautics, Beijing 100083, China

Abstract

Two different spatial levels are involved concerning damage accumulation to eventual failure. This can entail sample size $\ell$ ($\sim$ cm) to characteristic microdamage size $c*$ ($\sim$ µm). Associated are three physical processes with three different rates, namely macroscopic elastic wave velocity $a$, nucleation and growth rates of microdamage $n*$ and $V*$. It is found that the trans-scale length ratio $c*/\ell$ does not directly affect the process. Instead, two independent dimensionless numbers: the trans-scale one $D* = \frac{ac*}{(c*)^2/V*}$ and the intrinsic one $D^* = \frac{n*c*}{V*}$ including mesoscopic parameters only, play the key role in the process of damage accumulation to failure.

The above implies that there are three time scales involved in the process: the macroscopic imposed time scale $t_{im} = \ell/a$ and two meso-scopic time scales, nucleation and growth of damage, $t_n=1/(n*c^4)$ and $t_g=c*/V*$. Clearly, the dimensionless number $D* = t_g/t_{im}$ refers to the ratio of microdamage growth time scale over the macroscopically imposed time scale. So, analogous to the definition of Deborah number as the ratio of relaxation time over external one in rheology. Let $D^*$ be the imposed Deborah number while $D*$ represents the competition and coupling between the microdamage growth and the macroscopically imposed wave loading. In stress-wave induced tensile failure (spallation) $D^* < 1$, this means that microdamage has enough time to grow during the macroscopic wave loading. Thus, the microdamage growth appears to be the predominate mechanism governing the failure.

Moreover, the dimensionless number $D^* = t_g/t_n$ characterizes the ratio of two intrinsic mesoscopic time scales: growth over nucleation. Similarly let $D*$ be the “intrinsic Deborah number”. Both time scales are relevant to intrinsic relaxation rather than imposed one. Furthermore, the intrinsic Deborah number $D^*$ implies a certain characteristic damage. In particular, it is derived that $D^*$ is a proper indicator of macroscopic critical damage to damage localization, like $D^* \sim (10^{-3} - 10^{-2})$ in spallation. More importantly, we found that this small intrinsic Deborah number $D^*$ indicates the energy partition of microdamage dissipation over bulk plastic work. This explains why spallation can not be formulated by macroscopic energy criterion and must be treated by multi-scale analysis.

Keywords: Deborah numbers; Trans-scale coupling; Damage evolution; Failure.

\textsuperscript{*} Corresponding author.
E-mail address: why@lnm.imech.ac.cn (H.Y. Wang).
1. Introduction

From damage accumulation to failure, there prevails an important time-dependent phenomenon involving spallation in which failure occurs under transient loading like nano- to micro-seconds. Experimental observations suggest a time-integral criterion for spallation [1],

\[(\sigma/\sigma^* - 1) \times \Delta t = K\]  

where \(\sigma\) and \(\sigma^*\) are stress and a stress threshold respectively, \(\Delta t\) is the load duration, \(\nu\) and \(K\) are two parameters. This criterion indicates that the critical stress to spallation is no longer a material constant, but a variable depending on its loading duration. Furthermore, since the power exponent \(\nu\) in the criterion is usually neither 1 nor 2, the criterion implies neither momentum nor energy criteria macroscopically [1-3]. Comprehensive and critical reviews on spallation have been made [4-6]. It is stressed that “the continuum models based on the statistical nucleation and growth of brittle and ductile fracture appear to be an attractive approach, especially with a framework which provides some forms of a continuum cumulative-damage description of the evolving fracture state” [6]. Recently, the work in [7] suggests that “dynamic failure by the growth and coalescence of grain-boundary microcracks involve the cooperative interactions of propagating cracks. Insight into such processes is required from the perspective of stochastic mechanics and from computer simulations of the debonding of assemblages of grains”.

It follows that spallation is a typical process with coupled multiple space and time scales. At least, there are two length scales: the sample size at macroscopic level and the microdamage size at mesoscopic level. On the other hand, there are three time scales: the stress wave loading duration macroscopically, the mesoscopic nucleation time and growth time of microdamage. So, spallation represents an illustrative example with multiple space and time scales.

2. Statistical Microdamage Mechanics

The general evolution equation of microdamage number density is [8]

\[
\frac{\partial n}{\partial t} + \sum_{i=1}^{N} \frac{\partial (n \cdot P_i)}{\partial P_i} = n_N
\]  

where \(t\) is time, \(n_N\) is the nucleation rate of microdamage number density. \(P_i = \dot{P}_i\), “\(\cdot\)” denotes the rate of variable \(P_i\), which represents the state of microdamage. After taking the phase variable \(p\) as the current size of microdamage \(c\), i.e. \(p = c\), we have obtained a general solution to microdamage number density \(n(t,c; \sigma)\), [9-10]