VIBRATIONS OF DAMAGED 1D-3D MULTI-STRUCTURES

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Abstract. The objective of the paper is to study the vibrations of 1D-3D nondegenerate multi-structures, which are partly damaged. Such an analysis is useful to quickly assess the condition of a structure which has been in use over a long period of time. We look at three different positions of damage: top, middle or bottom of one leg. We calculate six eigenfrequencies associated with rigid body motion of the 3D structure and derive simple analytical asymptotic formulae for these eigenfrequencies. Hence, we analyse the effect of weakening of a leg. Accuracy of the asymptotic formulae is also checked with 3D finite element computations.

Key words: multi-structures, partial damage, asymptotics, eigenfrequencies

1. Introduction

A number of papers on eigenvalue problems posed for 2D-3D and 1D-3D multi-structures have been published in recent years (Aslanyan et al., 2002, 2003, 2003). These papers were motivated by the asymptotic study of static and spectral problems associated with 1D-3D multi-structures (Kozlov et al., 1999). More recently the transition between nondegeneracy and degeneracy in 1D-3D multi-structures has been studied (Aslanyan et al., 2005).

To our best knowledge, the only study on inhomogeneity in multi-structures is associated with high contrast mass densities between the 3D part and legs of a degenerate 1D-3D multi-structure (see the last chapter of Kozlov et al., 1999). The current problem is an extension to the study of structural damage on the vibrations of 1D-3D multi-structures (Aslanyan et al., 2005). We model the legs to have piecewise homogeneous sections. For the sake of simplicity, we consider only one of the legs to have this property.

2. General considerations for inhomogeneous nondegenerate multi-structures

Here, we first formulate an eigenvalue problem of 3D linear elasticity, as described in Kozlov et al., 1999, and discuss the general asymptotics for the skeleton (pile structure) of a 1D-3D nondegenerate inhomogeneous multi-structure.

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2.1. PROBLEM FORMULATION
We consider the following eigenvalue problem: \(-Lu = \rho \omega^2 u, \quad x \in \Omega \cup_{j=1}^{K} \Omega^{(j)}\), \([a^{(n)}(u)] = 0, \quad [u] = 0, \quad x \in S_{\text{int}}, \quad [a^{(n)}(u)] = 0, \quad x \in \partial \Omega_x \setminus S_x, \quad u = 0, \quad x \in S_x\), where \(L = \mu \nabla^2 + (\lambda + \mu) \text{grad} \, \text{div}\) is the Lamé operator (\(\lambda\) and \(\mu\) are the Lamé elastic moduli), \(u\) is the displacement vector, \([a^{(n)}(u)] = \sigma n\) is the stress vector, \(\rho\) is the mass density, \(\omega\) is the corresponding eigenfrequency and \(x = (x_1, x_2, x_3)\) are the Cartesian coordinates in \(\mathbb{R}^3\). Here \(\Omega\) is the 3D cap, \(\Omega^{(j)}, j = 1, 2, \ldots, K\) are the thin cylinders and \(S_x\) is the union of their base regions. \(S_{\text{int}}\) is used for all internal surfaces where the material parameters have discontinuities. These discontinuities are denoted by, for example, \([u]\) for the displacements. We assume that \(\lambda, \mu, \rho\) are constants in each subdomain.

2.2. BRIEF DESCRIPTION OF THE GENERAL ASYMPTOTICS
We assume that the 3D body moves like a rigid body to the leading order, i.e. \(v = \alpha + \beta \times x\) in \(\Omega\), where \(\alpha\) and \(\beta\) are constant vectors. We also assume that the displacements above are valid under the effect of body forces given by \(\Psi = c + d \times x\), where \(c\) and \(d\) are constant vectors. The dependency between the vectors \(v\) and \(\Psi\) can be established by obtaining the resultant force and the resultant moment (which appear in the equations of equilibrium) in terms of the vectors \(\alpha, \beta, c\) and \(d\) (see, for example, Aslanyan et al., 2003).

It was shown in Aslanyan et al., 2002 and 2003 that the spectral parameter associated with the skeleton is found by \(t \equiv \rho \omega^2 = \min_{\nu=0} \{ \int_{\Omega}(\nu, \Psi(\nu)) \, dx / \int_{\Omega}((\nu, \nu) \, dx) \}\), which can be reduced to the following characteristic equation: \(\det(\Phi - t \Gamma) = 0\). Here, \(\Phi\) and \(\Gamma\) are \(6 \times 6\) matrices; they are symmetric and positive definite, and all the roots of the characteristic equation above coincide with the positive definite matrix \(\Phi \Gamma^{-1}\). Hence, they are all positive: \(0 \leq t_1 \leq t_2 \leq \ldots \leq t_6\). Therefore, it is possible to approximate the values of the first six eigenfrequencies of the multi-structure \(\Omega_x\): \(\epsilon_k = \sqrt{\lambda_k / \rho} / (2\pi), k = 1, 2, \ldots, 6\). Here and in what follows \(\rho\) is associated with the 3D body.

3. A particular example: A 1D-3D multi-structure with six legs
Here we study some particular configurations of a 1D-3D multi-structure. In addition, we also consider different spacing between the neighbouring legs. To simplify the finite element modelling, the 3D body is chosen as a parallelepiped of dimensions \(a_1, a_2, a_3\). We assume that the legs are of length \(L\) and have \(b \times b\) cross-sections. They are assumed to be homogeneous with density \(\rho\), isotropic with Poisson’s ratio \(\nu\) and Young’s modulus \(E\), with the exception that the sixth leg is inhomogeneous, with Young’s modulus is given by:

\[ E(\gamma^{(6)}_3) = \{ E_1, \quad 0 < \gamma^{(6)}_3 < L_1, \quad E_2, \quad L_1 < \gamma^{(6)}_3 < L_2, \quad E_3, \quad L_2 < \gamma^{(6)}_3 < L \}. \] (1)

We introduce a dimensionless normalised parameter \(\epsilon = b / L\) (the characteristic ratio of the cross-section to the length of a leg) and use the following coordinates of the junction points: \(a^{(1)} = (a_1 / 2, 0, -a_3 / 2 + p_3), a^{(2)} = (a_1 / 2, 0, -a_3 / 2 - p_3), a^{(3)} = (a_1 / 2, 0, -a_3 / 2 + p_3)\).