Chapter 4

QUALITATIVE SPATIAL REASONING USING CONSTRAINT CALCULI

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1. Introduction

Qualitative reasoning is an approach for dealing with commonsense knowledge without using numerical computation. Instead, one tries to represent knowledge using a limited vocabulary such as qualitative relationships between entities or qualitative categories of numerical values, for instance, using $\{+, -, 0\}$ for representing real values. An important motivation for using a qualitative approach is that it is considered to be closer to how humans represent and reason about commonsense knowledge. Another motivation is that it is possible to deal with incomplete knowledge. Qualitative reasoning, however, is different from fuzzy computation. While fuzzy categories are approximations to real values, qualitative categories make only as many distinctions as necessary—the granularity depends on the corresponding application.

Two very important concepts of commonsense knowledge are time and space. Time, being a scalar entity, is very well suited for a qualitative approach and, thus, qualitative temporal reasoning has early emerged as a lively sub-field of qualitative reasoning which has generated a lot of research effort and important results. Space, in turn, is much more complex than time. This is mainly due

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to its inherent multi-dimensionality which leads to a higher degree of freedom and an increased possibility of describing entities and relationships between entities. This becomes clear when enumerating natural language expressions involving space or time. While temporal expressions mainly describe order and duration (like "before", "during", "long", or "a while") or a personal or general temporal category (like "late" or "morning"), spatial expressions are manifold. They are used for describing, for instance, direction ("left", "above"), distance ("far","near"), size ("large", "tiny"), shape ("oval", "convex"), or topology ("touch", "inside"). It is obvious that most spatial expressions in natural language are purely qualitative.

Although there are doubts that because of its multi-dimensionality, space can be adequately dealt with by using only qualitative methods (the poverty conjecture of Forbus et al., 1987), qualitative spatial reasoning has become an active research area. Because of the richness of space and its multiple aspects, however, most work in qualitative spatial reasoning has focused on single aspects of space. The most important aspects of space are topology, orientation, and distance. As shown in psychological studies (Piaget and Inhelder, 1948), this is also the order in which children acquire spatial notions. Other aspects of space include size, shape, morphology, and spatial change (motion).

Orthogonal to this view is the question for the right spatial ontology. One line of research considers points as the basic entities, another line considers extended spatial entities such as spatial regions as basic entities. While it is easier to deal with points rather than with regions in a computational framework, taking regions as the basic entities is certainly more adequate for commonsense reasoning—eventually, all physical objects are extended spatial entities. Furthermore, if points are required, they can be constructed from regions (Biacino and Gerla, 1991). A further ontological distinction is the nature of the embedding space. The most common notion of space is n-dimensional continuous space ($\mathbb{R}^n$). But there are also approaches which consider, e.g., discrete (Galton, 1999) or finite space (Gotts, 1996).

The most popular reasoning methods used in qualitative spatial reasoning are constraint based techniques adopted from previous work in temporal reasoning (see Sec. 2 for a comprehensive introduction to these techniques). In this chapter, we will focus exclusively on these techniques. In order to apply them, it is necessary to have a set of qualitative binary *basic relations* which have the property of being jointly exhaustive and pairwise disjoint, i.e., between any two spatial entities exactly one of the basic relations holds. The set of all possible relations is then the set of all possible unions of the basic relations. Reasoning can be done by exploiting *composition* of relations. For instance, if the binary relation R_1 holds between entities A and B and the binary relation R_2 holds between B and C, then the composition of R_1 and R_2 restricts the