

# Affine Invariant Feature Extraction Using a Combination of Radon and Wavelet Transforms

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**Abstract** - The paper presents a new framework for the extraction of region based affine invariant features with the view of object recognition in cluttered environments using the radon transform. The presented technique first normalizes an input image by performing data pre-whitening which reduces the problem by removing shearing deformations. Then four invariants are constructed by exploiting the properties of radon transform in combination with wavelets which enable the analysis of objects at multiple resolutions. The proposed technique makes use of an adaptive thresholding technique for the construction of invariants. Experimental results conducted using three different standard datasets confirm the validity of the proposed approach. Beside this the error rates obtained in terms of invariant stability in noisy conditions are significantly lower when compared to the method of moment invariants and the proposed invariants exhibit good feature disparity.

**Keywords** - Affine invariants, Radon transform, Feature Extraction, Geometric transformations, Wavelets, Pattern recognition.

## I. INTRODUCTION

Recognizing objects when subjected to perspective transformations in noisy and cluttered environments is one of the primary tasks in computer vision. Viewpoint related changes of objects can broadly be represented by weak perspective transformation which occurs when the depth of an object along the line of sight is small compared to the viewing distance [10]. This reduces the problem of perspective transformation to the affine transformation which is linear.

Constructing invariants to certain groups Euclidean, affine and projective deformations hold potential for wide spread applications for industrial part recognition [11], handwritten character recognition [12], identification of aircrafts [13], and shape analysis [14] to name a few.

The affine group includes the four basic forms of geometric distortions, under weak perspective projection assumption, namely translation rotation, scaling and shearing. Finding a set of descriptors that can resist geometric attacks on the object contour can act as a good starting point for the more difficult projective group of transformations.

In this paper we propose a new method of constructing invariants which is based on normalizing an affine distorted object using data pre-whitening which removes shearing distortion from the object on the Cartesian grid. Then a set of four invariants are constructed using a combination of radon

and wavelet transform over the region of support of the object and using the un-parameterized object boundary.

The rest of the paper is organized as follows. In section 2 we review some the previously published works, section 3 describes the proposed method in detail and section 4 provides experimental results and comparisons with previously published techniques. But before we move ahead let us have a brief overview of radon transform.

### A. Radon Transform

Introduced by Peter Toft [8] and related to Hough transform, it has received much attention in the past couple of years with applications emphasizing its use for line detection and localization [9]. Primarily it is able to transform two dimensional images with lines into a domain of possible line parameters where each line gives a peak at the corresponding orientation.

The two dimensional discrete radon transform for an images  $f(x,y)$  can be expressed as:

$$R(\rho, \theta) = \iint f(x,y) \delta(\rho - x \cos \theta - y \sin \theta) \quad (1)$$

where  $\rho$  is the distance of the line from the origin,  $\delta$  is the dirac delta and  $\theta$  is the orientation. Radon transform satisfies linearity, scaling, rotation and skewing which relates it directly to the affine group of transformations.

The above properties combined with the capability of radon transform to detect lines under high noise levels was the primary motivation for its selection as a tool for invariant feature extraction.

## II. RELATED WORK

Importance of constructing invariants to certain geometric transformations can be gauged from the fact that research has been conducted by many during the last two decades which can broadly be classified into two groups namely: Region based and Contour based invariant descriptors.

Region based techniques can further be classified into two groups: symmetric and asymmetric. In the context below we review some of the region based techniques that are most related to the present work.

Hu [1] introduced a set of seven affine moment invariants which were later corrected in [2] and have widely been used by the pattern recognition community. They are

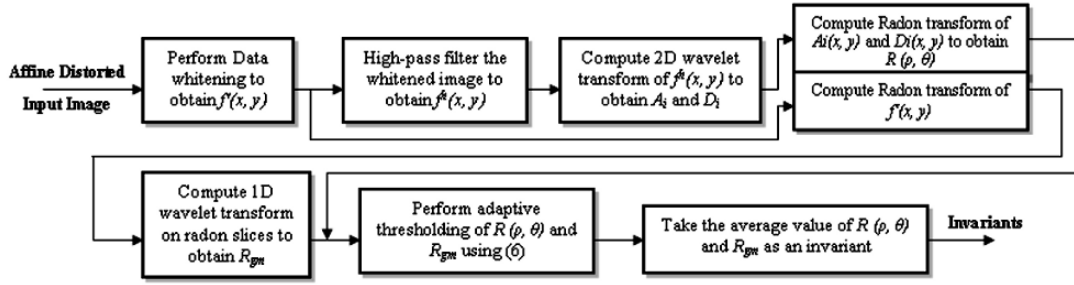


Fig. 1 shows the complete system diagram for the construction of region based invariant descriptors.

computationally simple and invariant to translation, scaling, rotation and skewing but suffer from several drawbacks like: information redundancy, which occurs because the basis used in their construction are not orthogonal, noise sensitivity, higher order moments are very sensitive to noise and illumination changes, finally large variation in the dynamic range of values may cause numerical instability with larger object size.

Some of the problems associated with moment invariants were addressed by Teague [3] who proposed the use of continuous orthogonal moments with higher expressive power. Zernike and Legendre moments were introduced by him based on the zernike and legendre polynomials. Zernike moments have proven to better represent object features besides providing rotational invariance and robustness to noise and minor shape distortions. But several problems are associated with the computation of zernike moments like numerical approximation of continuous integrals with discrete summations which leads to numerical errors affecting the properties such as rotational invariance and increase in computational complexity when the order of the polynomial becomes large.

Zhang et al [4] solved the problem of noise sensitivity associated with the framework of moment invariants and constructed invariants using the framework proposed in [2] in the spatial domain after Fourier filtering. An adaptive thresholding technique was developed as part of the framework to enable the establishment of the correspondence between the affine related images under high noise levels. But the technique has only been shown to work for symmetric images suffering from RST group of distortions.

More recently Petrou et al [5][6] introduced the trace transform for affine invariant feature extraction. Related to integral geometry and similar to radon transform however more general than either of them it computes image features along line integrals and performs calculations of any functional over the group of transformations in a global manner. They have used a set of three functionals namely line, diametrical and circus for the computation of invariant features. The major drawback is the computational cost which increases exponentially with the number of trace functionals.

Finally Heikkila et al [7] introduced the concept of autoconvolution across multiple scales of an input image and constructed a set of 29 invariants in the Fourier domain. They

use the expected value of autoconvolution as an invariant. Although their technique produces excellent results but their method results in feature overlapping across different frequencies which serves as a major limitation in object classification.

In an attempt to solve the problems mentioned above, the present work makes use of the radon transform in combination with wavelets to construct a set of invariants that can be used for recognizing objects under the affine transformations coupled with high noise distortion levels.

In short we improve on many of the short comings mentioned above.

### III. PROPOSED TECHNIQUE

We propose a three step process for the construction of region based invariant descriptors of the objects. The first step acts as foundation for second and third steps in which radon transform is applied and then invariants are constructed. In the context below we provide the detailed description of each step.

#### A. Data Pre-whitening

Let us consider an image  $f(x, y)$  which is parameterized as  $Y(t) = [x(t), y(t)]$  with parameter  $t$  on a plane by performing raster scanning on the coordinate axis. Thus a point from  $Y(t)$  under an affine transformation can be expressed as:

$$\begin{aligned}\tilde{x}(t) &= a_0 + a_1x(t) + a_2y(t) \\ \tilde{y}(t) &= b_0 + b_1x(t) + b_2y(t)\end{aligned}\quad (2)$$

The above equations can be written in matrix form as:

$$\begin{aligned}\begin{bmatrix} \tilde{x}(t') \\ \tilde{y}(t') \end{bmatrix} &= \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} + \begin{bmatrix} a_0 \\ b_0 \end{bmatrix} \\ \begin{bmatrix} \tilde{x}(t') \\ \tilde{y}(t') \end{bmatrix} &= P \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} + B \\ Y'(t') &= PY(t) + B\end{aligned}\quad (3)$$

Next whitening is performed on  $Y'(t')$  by computing the Eigen value decomposition of covariance matrix as: