Numerical Solution to Horizontal Zero-inertia, Viscous Dam-Break Problem

Blaise NSOM, Mathieu JENNY, Youcef ASKOURA and Frédéric PERON
Université de Bretagne Occidentale -. LIME / IUT de Brest. BP 93169. Rue de Kergoat. 29231 BREST Cedex 3 - France blaise.nsom@univ-brest.fr

Abstract : Debris flows such as avalanches and lahars differ from the classical dam-break problem of hydraulics due to the relative importance of viscous versus inertial forces in the momentum balance. An equation of motion describing debris flow in the limit of zero inertia is developed and solved using a converged finite difference numerical, in two limits: short time and long time. These solutions are then combined into a single, universal model.

I. Introduction

Since Ritter's original work on dam-break flow [1], many studies have been performed focusing on experiments, theory and numerical methods (Gill [2]). Dam-break flow has become a classical hydraulic problem with such a large complexity that a higher degree of reproduction of real conditions raises new studies, as certain scenarios of initiation of debris flows, flash floods and lahars can be modelled by dam failures. So, among others, Zanuttigh and Lamberti [3] apply an exact Riemann solution that allows a second-order accuracy of the solution for the power-law section shape to the dam-break problem in valleys with different shapes but the same dam area; Frazao and Zech [4] present an experimental study of a dam-break flow in an initially dry channel with a 90° bend, and successfully compare their measurements of water level and velocity field with numerical results.
Consider a dam obstructing a horizontal smooth channel, dry downstream and with a given quantity of fluid upstream (with height h_0), contained between a fix plate and a dam.

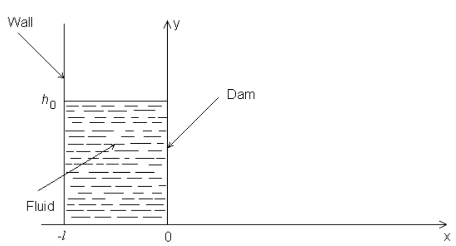

Fig.1: Flow Configuration at negative time

At initial time, the dam collapses and the fluid is released downstream (positive wave), while a negative wave propagates upstream (negative wave). From dam-collapse date to time where negative wave reaches the fix plate, Ritter [1] gives the so-called inertial solution, stating that the wave front advances with a constant speed of $2\sqrt{gh_0}$, while the negative wave moves back with constant speed $\sqrt{gh_0}$.
Between these two extremities, average speed U and the hydrograph are respectively given by

$$U=\frac{2}{3}\left(\frac{x}{t}+\sqrt{gh_0}\right) \tag{1}$$

$$\sqrt{gh}=\frac{1}{3}\left(2\sqrt{gh_0}-\frac{x}{t}\right) \tag{2}$$

where the dam is assumed to be located at $x=0$.
This configuration generally represents flow generated by dam failure caused by exceptional rainfall (e.g. Malpasset, France in 1959) or by war action (e.g. Dnieproghes, Ukraine in 1941). The fluid is water and the flow is described by the Navier Stokes and continuity equations, together with the non slip condition. Assuming the shallow water approximation, this system of equations leads to the Barré de Saint-Venant equations (De Saint Venant [5]), a one-dimensional hyperbolic system. The complete hydrodynamic equations describing this unsteady flow in open channel were solved by Faure and Nahas [6], using the method of characteristics. Hunt [7], comparing one-dimensional turbulent flow model down a slope with its viscous counterpart, concluded that the viscous flow model gives the best description for debris flows. Indeed, these flows develop within a long domain, i.e. a domain of space that is much longer than it is wide, so short time behavior described by the previous studies are inappropriate to give a complete description of these natural flows. In the nature, the fluid is generally mud, i.e. a very viscous complex mixture of water with diverse sediments, so the viscous terms are dominant here over the inertial ones. To represent such natural dam-break flow, Nsom et al. [8], Nsom [9] performed an experimental study with glucose-syrup fluids characterized with adjustable viscosity and density. Hunt [7] built similarity solutions for "geological flows" down a sloping 1D channel. Also, Schwarz [10] achieved a numerical study of viscous thin liquid films down an inclined plane. Solving free surface lubrication equations, including the effects of both gravity and surface tension, he states a scaling law for the prediction of finger-width.
In this work, a 1-D model is presented, aiming to provide practical laws, useful to engineers. A priori knowledge of the speed of the flood wave is indeed important because this will determine the available time in which forecast and rescue measures need to be effected. Assuming the shallow-water approximation, equations of motion governing viscous dam-break flow are built and put in non-dimensional form and the initial and boundary conditions are stated. Then, an analytical solution is presented both for short time and long time behavior. Zoppou and Roberts [11] tested the performance of 20 explicit schemes used to solve the shallow water wave equations for simulating the dam-break problem. Comparing results from these schemes with analytical solutions to the dam-break problem with finite-water depth and dry bed downstream of the dam, they found that most of the numerical schemes produce

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reasonable results for subcritical flows. So an explicit procedure was used here, which does not take into account turbulence generated by dam-break wave, as the flow develops over a dry smooth bed (Shigematsu et al. [12]). Numerical results are shown and compared with the analytical ones in each regime.

II. Problem statement

Equations of motion

Let h_0 denote the height of fluid at negative time in a smooth horizontal rectangular channel, g the gravity, ρ and μ the fluid density and viscosity, respectively. Using a cartesian system of coordinates with origin at dam site, x-axis lying on channel-length and z-axis in upwards vertical direction (fig. 1). The fluid is assumed to flow mainly in the direction of x-axis with height h at given control section of abscisse x, at time t. So, the vertical velocities are negligibly small, and therefore the pressure is hydrostatic, the pressure in the flow is given by

$$p = p_0 + \rho g(h - z) \tag{3}$$

where p_0 denotes the (constant) pressure at the free surface and g the gravity. The balance between the pressure gradient and the viscous forces is thus expressed by

$$\frac{1}{\rho}\frac{\partial p}{\partial x} = g\frac{\partial h}{\partial x} = \nu\frac{\partial^2 u}{\partial z^2} \tag{4}$$

where horizontal derivatives have been neglected in comparison with vertical derivatives on the right-hand side of (4) because the length of the current is very much greater than its thickness. At the base of fluid layer the no slip condition writes

$$u\left(x,0,t\right) = 0 \tag{5}$$

Considering that the shear stress at the top of the current is very much less than its value within the current, and then can be approximated as

$$\frac{\partial u}{\partial z}\left(x,h,t\right) = 0 \tag{6}$$

the solution of (4), (5) and (6) is

$$u\left(x,z,t\right) = -\frac{1}{2}\frac{g}{\nu}\frac{\partial h}{\partial x}z\left(2h - z\right) \tag{7}$$

A complete determination of the unknowns u and h requires the equation of continuity which can be written here as

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x}\left(\int_0^h u\,dz\right) = 0 \tag{8}$$

Substituting (7) into (8) we obtain

$$\frac{\partial h}{\partial t} - \frac{\rho g}{12\mu}\frac{\partial^2 (h^4)}{\partial x^2} = 0 \tag{9}$$

If l denotes the reservoir length, assume the following set of non dimensional variables:

$$(h', x', x_f', t') = \left(\frac{h}{h_0}, \frac{x}{h_0}, \frac{x_f}{h_0}, \frac{\rho g h^3}{12\mu l^2}t\right) \tag{10}$$

where subscript f denotes wave-front, equation of motion (9) then becomes, in non dimensional form:

$$\frac{\partial^2 (h'^4)}{\partial x^2} - \frac{\partial h'}{\partial t'} = 0 \tag{11}$$

Eq.11 is similar to the equation of motion obtained by Schwarz [10] and Barthes-Biesel [13], in describing the evolution of a thin liquid layer, flowing down a horizontal plane when surface tension effects can be neglected.

Initial and boundary conditions

Using (10), the fluid height at initial time is given by:

$$h'\left(x', t' = \bar{0}\right) = 1 \qquad \text{for} \qquad -1 \le x' \le 0 \tag{12}$$

$$h'\left(x', t' = \bar{0}\right) = 0 \qquad \text{otherwise} \tag{13}$$

Furthermore, a complementary boundary condition should be imposed upstream, assuming that a short time or an asymptotic solution is sought. These boundary conditions are suggested by experimental observation. For short time case, it is written:

$$h'\left(x' = -l', t'\right) = 1 \qquad \text{with} \qquad l' = \frac{l}{h_0} \tag{14}$$

which means that only a given fluid quantity in the upper part of the reservoir is released downstream at the very first instants following dam collapse. While for long time case, it is written:

$$\frac{\partial h'}{\partial x'}\left(x' = -l', t'\right) = 0 \tag{15}$$

which means that there is no flow at the fixed wall so, at that site, the free surface is horizontal.

For convenience, in the rest of the paper, non dimensional quantities will be denoted by corresponding capital letters with no primes.

III. Numerical solution

Discretization

To build a numerical procedure, it is necessary to define the channel total length l_t. The non dimensional extreme (downwards) abscissa is

$$L_e = \frac{l_t - l}{h_0} \tag{16}$$

This point is so far from dam site, that the flow is supposed to never reach it during the while of a given experiment (1D assumption), with total duration τ. Then, the problem to solve numerically is described by the following equation of motion

$$\frac{\partial H}{\partial T} = \frac{\partial^2 (H^4)}{\partial X^2} \tag{17}$$

associated with the following initial conditions

$$H(X,0) = 1 \qquad \text{if} \qquad X \in \left[-L, 0\right] \tag{18}$$

$$H(X,0) = 0 \qquad \text{otherwise} \tag{19}$$

and boundary conditions

$$\frac{\partial H}{\partial X}(-L, T) = 0 \qquad \forall T \ge 0 \tag{20}$$

$$H(Xe, T) = 0 \qquad \forall T \ge 0 \tag{21}$$

This problem is solved by a finite difference method.

For this, the function $H(X,T)$ is computed in the set $\left[-L, Le\right] \times \left[0, \tau\right]$, itself discretized in a finite number of identical small rectangles with sides ΔT and ΔX.

The equation will be approximated at grid points located at the following coordinates in the $\left[-L, Le\right] \times \left[0, \tau\right]$ set:

$$\left(X_i, T_j\right) = \left(-L + i \cdot \Delta X, j \cdot \Delta T\right), \; i \in \left[0, \frac{-L + Le}{\Delta X}\right], \; j \in \left[0, \frac{\tau}{\Delta T}\right] \tag{22}$$

Notice that eq.(32) can be put in the form