Describing Function and Error Estimation for Class of Nonlinear Systems with Fuzzy elements

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Abstract **– The procedure for approximate analytical determination of describing function of nonlinear systems with odd static characteristics is presented in the paper. Generalized mathematical expressions for determining such describing function with error estimation are given. The procedure is illustrated on determination of describing function, and corresponding error estimation, of Mamdani fuzzy element and nonlinear system with fuzzy element and saturation nonlinearity.**

I. Introduction

Describing function is an equivalent gain of nonlinear element, defined by the harmonic linearization method of nonlinear static characteristic [4, 7 and others]. It is a known method of analysis and synthesis when nonlinear system can be decoupled into linear and nonlinear parts (Fig.1). If the linear part of the system has the characteristics of low-pass filter and if we apply periodical signal to the system, output signal will have the same base frequency as input signal with damped higher frequencies.

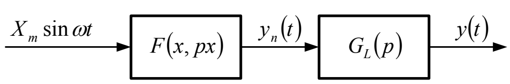

Fig.1 Nonlinear system represented with decoupled nonlinear $F(x, px)$ and linear parts $G_L(p)$, $p = d/dt$.

If the amplitudes of higher harmonics are relatively small to the amplitude of the first harmonic, output signal can be approximated by its first harmonic. Mathematically, first harmonic of the output signal, for the sinusoidal input signal $X_m \sin\omega t$, can be expressed by the Fourier expressions:

$$\begin{aligned} y_N(t) &\approx Y_{P1}\sin\omega t + Y_{Q1}\cos\omega t \\ y_N(t) &\approx \mathrm{Im}\left\{\left(Y_{P1} + jY_{Q1}\right)e^{j\omega t}\right\} \end{aligned} \tag{1}$$

$$Y_{P1} = \frac{1}{\pi}\int_0^{2\pi} F\left(X_m \sin\omega t\right)\sin\omega t \, d(\omega t) \tag{2}$$

$$Y_{Q1} = \frac{1}{\pi}\int_0^{2\pi} F\left(X_m \sin\omega t\right)\cos\omega t \, d(\omega t) \tag{3}$$

where Y_{P1} and Y_{Q1} are first Fourier coefficients. Describing function is the ratio between first harmonic of the output signal and input signal in complex form:

$$G_N(X_m) = P(X_m) + jQ(X_m) = \frac{Y_{P1}}{X_m} + j\frac{Y_{Q1}}{X_m} \tag{4}$$

where $P(X_m)$ and $Q(X_m)$ are coefficients of the harmonic linearization [4, 7 and others]. Determination of describing function boils down to the determination of integral expressions (2, 3) for the known static characteristic of the nonlinear part of the system. If the static characteristic of nonlinear system cannot be analytically expressed or integral expressions can not be solved, describing function can be determined by experimental (simulation) method [1, 2] or by some method of numerical integration. In that case, the result is more often than not in form of graphical record of the experiment or simulation and the problem of determining the mathematical description of describing function arises. In the case of fuzzy systems, especially of Mamdani type, there is yet no good method for stability analysis. Use of describing function allows for such analysis by using well known and developed procedures ([4], [7] and others). In the case of fuzzy systems there was some work on analytical determination of describing function of fuzzy controllers. For example, analytical approach to determining fuzzy controller describing function is given in [9]. However, the obtained describing function is very complicated. Moreover, for the procedure to be applied a symmetrical fuzzy controller with triangular membership functions is required. In [1] an experimental method for the determination of the SISO fuzzy controller describing function by computer simulation is described. An experimental evaluation of the fuzzy controller describing function is also given in [8]. However, the

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procedure is not given in detail and the analysis is conducted for the system with the only one nonlinearity, thus only for a linear system controlled by the fuzzy controller. The method described in this work can be used to determine describing function of nonlinear systems with odd static characteristics, and is illustrated on Mamdani PD fuzzy element and Mamdani PD fuzzy element in series with saturation. The paper is organized as follows. In section II a method for approximation of static characteristic is given. Section III illustrates error estimation of approximation. Sections IV and V illustrate the procedure on example and Section VI gives the conclusion.

II. Determination of Describing Function Using Linearly Approximated Static Characteristic

Arbitrary nonlinear static characteristic can be approximated by the linear piecewise elements. Integral expressions (2, 3) for the describing function will be than expressed by an approximated analytical solution. General solution for the nonlinear odd static characteristic with hysteresis, approximated by the linear piecewise elements is presented in [3]. For the odd functions the analysis can be conducted in the first quadrant only. The approximation of the static characteristic $F_i(x)$ is shown in Fig. 2. Linearly approximated static characteristic is defined by two sets of points, rising and falling part of the static characteristic:

$$\left[g_i, F(g_i)\right],\ i = 1,\dots,G \tag{5}$$

$$\left[h_i, F(h_i)\right],\ i = 1,\dots,H \tag{6}$$

where set (5) defines rising part and set (6) defines falling part of the characteristic.

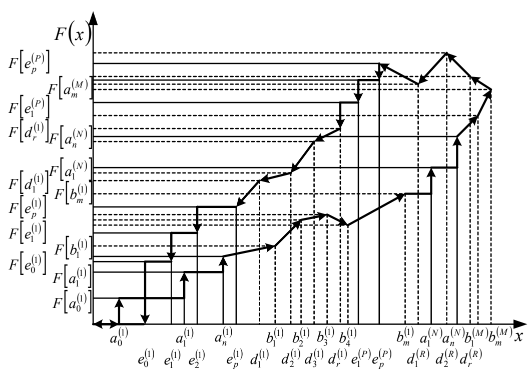

Fig.2 Static characteristic approximated with linear piecewise elements.

In order to simplify the presentation of the final expressions, sets (5) and (6) are further divided into groups of two types of linear elements with respect to the first derivative, step elements ($F'(x) = 0$ or ∞) and slope elements ($F'(x)\#0$ and finite). Each point of the set is designated with two indexes; the upper index denotes the order number of the group; the lower index denotes the order number of the element within the group. Rising part of static characteristic is defined by:

$$\left[a_i^{(j)}, F\left(a_i^{(j)}\right)\right],\ j = 1,\dots,N,\ i = 1,\dots,n(j) \tag{7}$$

$$\left[b_i^{(j)}, F\left(b_i^{(j)}\right)\right],\ j = 1,\dots,M,\ i = 1,\dots,m(j) \tag{8}$$

where pairs $\left[a_i^{(j)}, F\left(a_i^{(j)}\right)\right]$ denote position of step group points, pairs $\left[b_i^{(j)}, F\left(b_i^{(j)}\right)\right]$ denote position of slope group points, N is the number of step groups, $n(j)$ is the number of step elements per each group, M is the number of slope groups and $m(j)$ is the number of slope elements per each group. Falling part of the static characteristic is defined by:

$$\left[e_i^{(j)}, F\left(e_i^{(j)}\right)\right],\ j = 1,\dots,P,\ i = 1,\dots,p(j) \tag{9}$$

$$\left[d_i^{(j)}, F\left(d_i^{(j)}\right)\right],\ j = 1,\dots,R,\ i = 1,\dots,r(j) \tag{10}$$

where pairs $\left[e_i^{(j)}, F\left(e_i^{(j)}\right)\right]$ denote position of step group points, pairs $\left[d_i^{(j)}, F\left(d_i^{(j)}\right)\right]$ denote position of slope group points, P is the number of step groups, $p(j)$ is the number of step elements per each group, R is the number of slope groups and $r(j)$ is the number of slope elements per each group. Dividing the integral expressions (2, 3) into integral parts over the rising and falling parts of the static characteristic, and substituting integrals with the sum of integrals over the linear elements, we obtain the following algebraic expressions ([3]) of coefficients of the harmonic linearization:

$$P(X_m) = P_U(X_m) + P_S(X_m) \tag{11}$$

$$\begin{aligned}
P_U(X_m) = \sum_{j=1}^{M} \Bigg\{ & \frac{K_m^{(j)}}{\pi} * \\
* & \left(\arcsin\frac{b_m^{(j)}}{X_m} - \frac{b_m^{(j)} - 2b_{m-1}^{(j)}}{X_m}\sqrt{1 - \frac{b_m^{(j)2}}{X_m^2}} \right) - \\
& - \frac{K_1^{(j)}}{\pi}\left(\arcsin\frac{b_0^{(j)}}{X_m} + \frac{b_0^{(j)}}{X_m}\sqrt{1 - \frac{b_0^{(j)2}}{X_m^2}} \right) - \\
& - \frac{2F\left(b_0^{(j)}\right)}{\pi X_m}\left(\sqrt{1 - \frac{b_m^{(j)2}}{X_m^2}} - \sqrt{1 - \frac{b_0^{(j)2}}{X_m^2}} \right) - \\
& - \frac{1}{\pi}\sum_{i=1}^{m-1}\Bigg[\left(K_{i+1}^{(j)} - K_i^{(j)}\right)\left(\arcsin\frac{b_i^{(j)}}{X_m} + \frac{b_i^{(j)}}{X_m}\sqrt{1 - \frac{b_i^{(j)2}}{X_m^2}} \right) + \\
& + \frac{2K_i^{(j)}}{X_m}\left(b_i^{(j)} - b_{i-1}^{(j)}\right)\sqrt{1 - \frac{b_m^{(j)2}}{X_m^2}} \Bigg] \Bigg\} + \\
& + \frac{2}{\pi X_m}\sum_{j=1}^{N}\Bigg\{ F\left(a_0^{(j)}\right)\sqrt{1 - \frac{a_0^{(j)2}}{X_m^2}} - F\left(a_n^{(j)}\right)\sqrt{1 - \frac{a_n^{(j)2}}{X_m^2}} + \\
& + \sum_{i=1}^{n}\left(F\left(a_i^{(j)}\right) - F\left(a_{i-1}^{(j)}\right)\right)\sqrt{1 - \frac{a_i^{(j)2}}{X_m^2}} \Bigg\}
\end{aligned} \tag{12}$$