Nonlinear Time Invariant Systems

7.1 Introduction

Not all electronic problems can be solved by linear systems. Many capabilities possess the nonlinear ones, which output and input are coupled nonlinearly. Even if the problem is soluble with an LTV system, the nonlinear device does it usually with a simpler structure; in some cases just with several units.

If a nonlinearity does not undergo any changes with time, the system is nonlinear time-invariant (NTI). In control systems, nonlinearity is typically required of the certain shape and can even be synthesized. To design quadratic and logarithmic amplifiers, semiconductor components are used providing the necessary characteristics. In many cases, piecewise nonlinearities meet practical needs. Let us add that, in all electronic systems, natural spurious nonlinearities occur owing to saturation.

In SISO NTI systems, the input \( y(t) \) and output \( x(t) \) are coupled by the nonlinear time-invariant operator \( \mathcal{O}(x) \),

\[
y(t) = \mathcal{O}(x) x(t) \equiv \mathcal{O}[x(t)] x(t),
\]

that can be represented by the nonlinear ODE, integro-differential equation, or integral equation. To study dynamics and stability, the qualitative methods are used universally. Rigorous analytical solutions of nonlinear problems are usually available only in particular cases. On the other hand, in the overwhelming majority of practical situations, of importance are solutions for harmonics of the input. In view of that, different variations of the methods of averaging and linearization are used rendering a tremendous influence on the engineering theory of nonlinear systems.

Our discussion of linear systems was started with the convolution and differential equation, so with memory (dynamic) solutions. This is because the memoryless linear system is nothing more than a time-invariant or time-varying gain. In memoryless nonlinear systems, the gain depends on the signal, therefore such systems need special investigations. If a nonlinear system is
memory, the gain inherently changes with time taking different values for the same signal.

Fig. 7.1 sketches a typical picture explaining how memory affects the output of an NTI system. Suppose that the input-to-output dependence $y[x(t)]$

is nonlinear. If $x(t)$ changes with time slowly, a circular transition from the point $A$ to $B$ would trace along the same trajectory (“Slow” time). With fast changes, any dynamic system inherently demonstrates inertia and the transitions from $A$ to $B$ and back to $A$ do not coincide in trajectories (“Fast” time). The effect is called *hysteresis*. The negative appearance of hysteresis is that the input produces two values of the output with coordinates dependent on the input rate.

Of course, the terms “slow” and “fast” are conditional in a sense. Conventionally, one may think that if the spectrum of an input signal corresponds to the “slow” time of a system, then the latter is memoryless (negligible hysteresis). Having no memory, a system is described by $y(x)$ that does not involve time $t$ as a variable. Otherwise, a system is memory, $y[x(t)]$ or (7.1).

**7.2 Memoryless Systems**

If a SISO NTI system is memoryless, its input is coupled with the output by the operator (7.1) losing a variable $t$ such that

$$y = \mathcal{O}(x)x = y(x).$$

(7.2)