Chapter 10

New trends: Network fibring

In the previous chapters we investigated different aspects of fibring, always in a clear logical context. That is, we always assumed that the original components were logics with certain characteristics. Now we are faced with a new problem. There are application domains where we seem to have a fibring situation but the problem is initially presented with networks instead of logic systems. One of the messages is that if we look at the problem with networks seen as labeled deductive systems then we immediately are in a logical context. In this way, we broaden substantially the applications of fibring to many unusual domains like neural networks in bioinformatics and argumentation theory. The chapter starts with four case studies illustrating several areas to which we want to extend fibring. In many cases recursive networks are used. The notion of self-fibring of networks is a general abstraction where we can accommodate these different fibring situations.

In Section 10.1, we motivate network fibring using a labelled formulation of modal logic. Next, in Sections 10.2, 10.3, 10.4 and 10.5, we introduce some case-studies. Section 10.2 discusses integration of information flows and describes a system in which reasoning and proofs from different sources of information can be accommodated. In Section 10.3, we refer to some generalizations of logic input/output operations. We also discuss how to combine input/output operations into networks. In Section 10.4, we discuss the fibring of neural networks. In Section 10.5, we turn our attention to recursive Bayesian networks. In Section 10.6, we give the notion of self-fibring of networks. Finally, Section 10.7 presents some concluding remarks.

This chapter capitalizes on the following works: [111] for network modalities, [110] for integrating flows of information using LDS, [79] for neural networks and [275] for Bayesian networks.
1.1 Introduction

The applications of fibering seem to go beyond the case where we want to combine two logics. Examples can be found in such contexts as bioinformatics and argumentation theory. It seems that the most general concept that can be used as the framework for fibering are networks seen as labeled deductive systems [105].

The theory of labeled deductive systems (LDS) was developed from the bottom up point of view, especially to model aspects of human behavior, reasoning and action, and is very comprehensive, adaptable and incremental. It contains a large variety of existing logical systems as special cases. LDS is not a single system but a methodology for building families of systems, ready to be adapted to the needs of various application areas.

In this section, we motivate network fibring by looking essentially at a labeled formulation of modal logic with language $L$. Herein, we assume that the semantics is presented by a Kripke structure of the form

$$m = (W, R, w, \models)$$

where, besides the non-empty set of possible worlds $W$ and the accessibility relation $R \subseteq W^2$, we consider an actual world $w \in W$ and a satisfaction relation $\models \subseteq W \times L$. We consider a satisfaction relation instead of a valuation because we want to stress that the exact recursive definition of satisfaction is not relevant to the discussion. We have that

$$m \models \varphi \text{ whenever } w \models \varphi.$$

Given a class $M$ of models we can define a semantic consequence relation as follows. We say that a formula $\gamma$ entails a formula $\varphi$ and write

$$\gamma \models \varphi$$

if $m \models \varphi$ for every model $m \in M$ such that $m \models \gamma$.

For the purposes that we have in mind, we need to extend the signature with labels. Labels represent the worlds at the syntactic level. We also need a way to represent $R$, the accessibility relation, at the syntactic level.

As a consequence, besides the usual modal formulas we also have formulas relating the labels like $aRt$ or $\langle a, t \rangle$, where $a$ and $t$ are labels and labeled formulas like $a: \varphi$ meaning that we want to state that $\varphi$ is true at $a$.

A network specification is a triple

$$\langle S, F, f \rangle$$

where:

- $S$ is a set of labels (representing worlds at the syntactic level);
- $F$ is a set of formulas of the kind $\langle t_1, t_2 \rangle$ (representing the accessibility relation at the syntactic level);